Beam Loading Effects in photo-injectors. Simulations and Measurements

Longitudinal Electron beam Dynamics for coherent light Sources

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03.10.2023

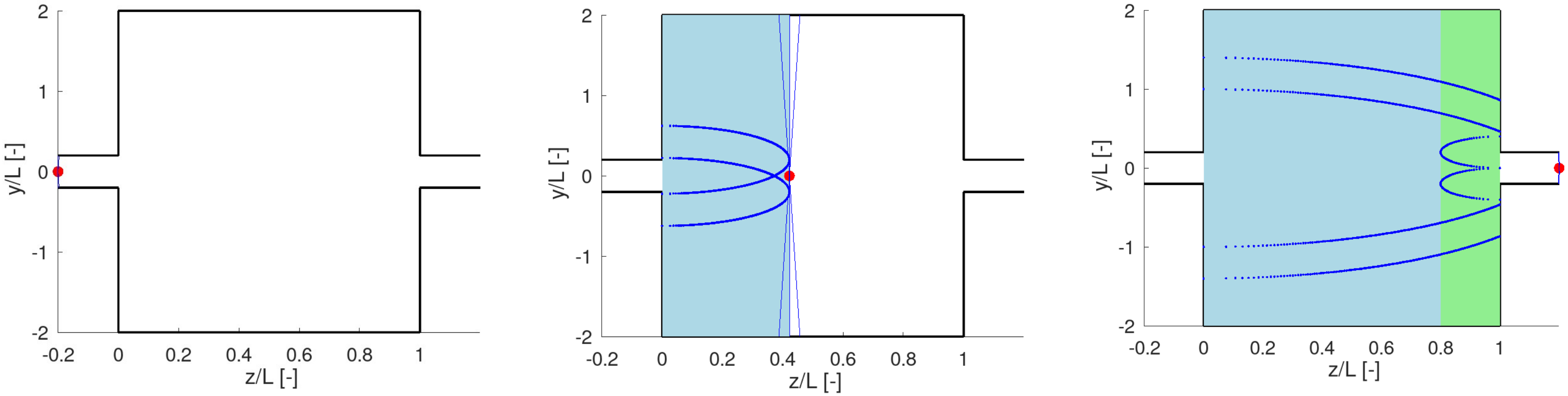
Outline

- **PART I**: Introduction. The Beam Loading Effect
  - TW model
- **PART II**: BL in Standing-Wave Photo-injectors. Particularities
- **PART III:** BL Simulations with RF-Track
- **PART IV**: Results
  - Simulations vs Measurements in the CLEAR facility
  - BL Compensation

PART I: The Beam Loading Effect

Beam Loading Effect

- Beam – Cavity interaction: **Excitation** of the **fundamental accelerating mode**
  - Beam-induced field causes **deceleration**



> Animation of a particle flying through a cavity and leaving an EM field behind it.

Beam Loading Effect

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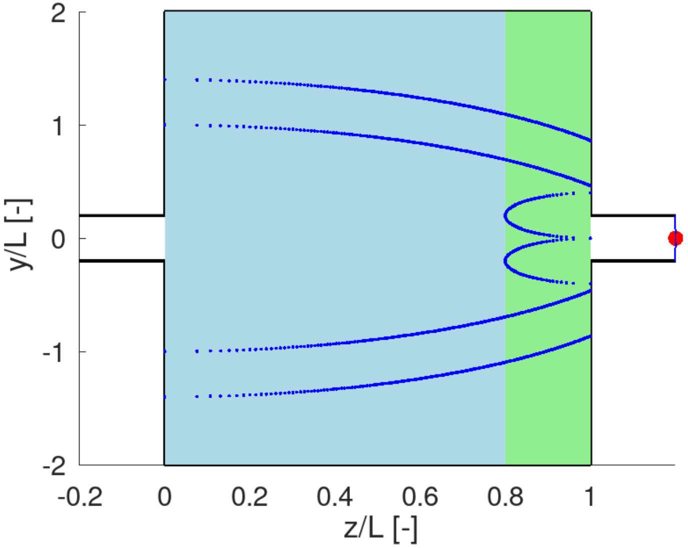
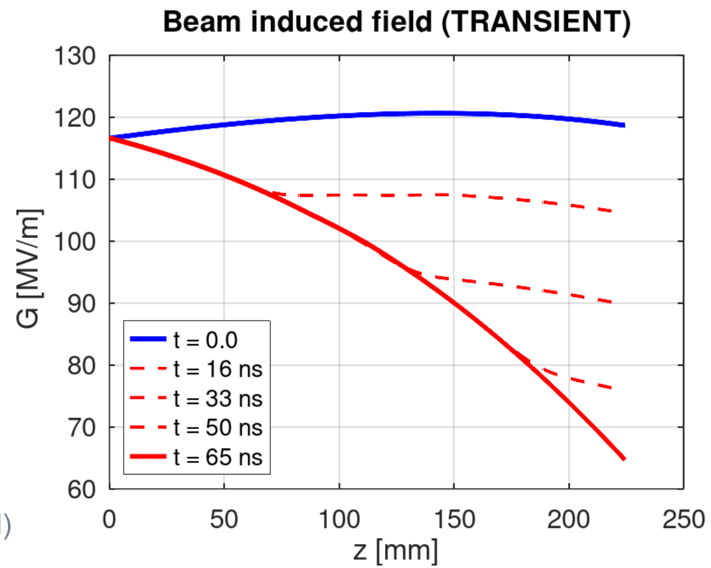


- The induced excitation lasts for a long time
  - **Long range effect**
  - Accumulated from bunch-to-bunch

> Animation of a particle flying through a cavity and leaving an EM field behind it.

Beam Loading Effect

- **What**: Reduction of available accelerating gradient
- **Origin**: Beam – Cavity interaction
- **Consequences**: Transient response
  - Different energy loss from bunch to bunch
- **Motivation**: High I, Compact accelerating structures



> Theoretical analysis of beam loading effect based on CLIC's main linac [2]

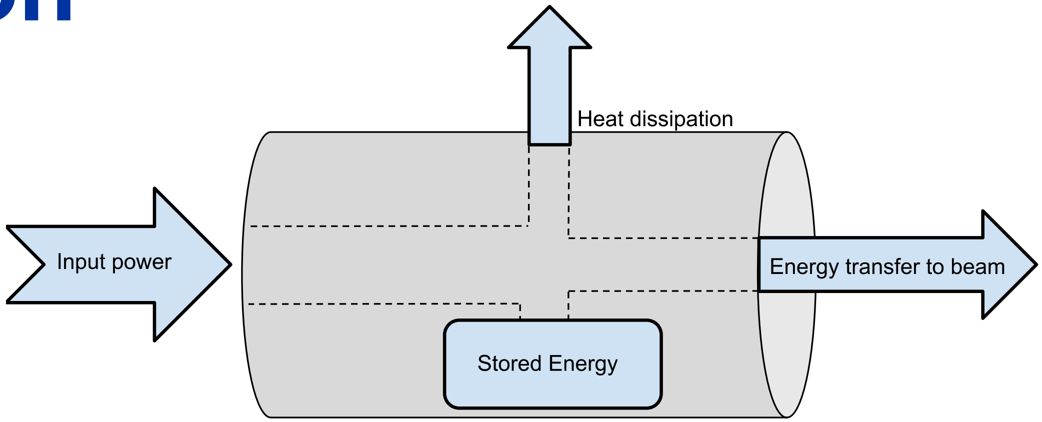
[1] P. Lapostolle. *Linear Accelerators*. North Holland Publishing Company, 1970 (Amsterdam, Holland)

[2] A. Grudiev, A.Lunin, V. Yakovlev. *Analytical solutions for transient and stead state beam loading in arbitrary travelling wave accelerating structures.* Phys. Rev. Special topics **14**, 052001 (2011)

I. Energy Conservation

- Poynting Theorem

$$-\underbrace{\frac{\partial u(\vec{r},t)}{\partial t}}_{\text{Stored EM energy density variation}} = \underbrace{\vec{\nabla} \cdot \vec{S}(\vec{r},t)}_{\text{Power Flow \& Loss}} + \underbrace{\vec{E}(\vec{r},t) \cdot \vec{J}(\vec{r},t)}_{\text{Field-Beam Interaction}}$$



> Energy balance schematics for an accelerating structure

- Figures of merit:
  - Group velocity $$v_g = \frac{P_{\text{flow}}}{w} \,[\text{m/s}]$$
  - Quality factor $$Q = \omega_{\text{RF}} \frac{w}{p_{\text{diss}}}$$
  - Shunt impedance (p.u.l) $$r_e = \frac{G_{\text{eff}}^2}{p_{\text{diss}}} \,[\Omega/\text{m}]$$

[3] Thomas P. Wangler. *RF linear accelerators*. Wiley-VCH 2008 (Amsterdam, Holland)

I. Gradient description

- **Gradient**: Averaged E-field *affecting* the particle $t_f(z) \simeq \frac{z}{c}$

TW wave $E_z(z,t) = \mathrm{Re}[E_0(z,t)e^{j(kz-\omega t)}] \xrightarrow{\text{leads to}} G(z,t)$

SW wave $E_z(z,t) = \mathrm{Re}[E_0(z,t)e^{j\omega t}] \xrightarrow{\text{leads to}} G_{\text{eff}}(z,t)$

- Time Transit factor

$$\mathcal{T}(z,t) = \frac{G_{\text{eff}}(z,t)}{G(z,t)} = \frac{\frac{1}{L}\int_0^L \mathrm{Re}[E_0(z,t)e^{j\omega t(z)}]\mathrm{d}z}{\frac{1}{L}\int_0^L |E_z(z,t)|\mathrm{d}z}$$

[3] Thomas P. Wangler. *RF linear accelerators*. Wiley-VCH 2008 (Amsterdam, Holland)

I. Power-Diffusion PDE

- From Poynting: Equation in terms of Gradient:

$$-\frac{\partial G_{\text{eff}}}{\partial t} = v_g \frac{\partial G_{\text{eff}}}{\partial z} + \left(-\frac{v_g Q}{r_{\text{eff}}} \frac{\partial (r_{\text{eff}}/Q)}{\partial z} + \frac{\omega}{Q} + \frac{\partial v_g}{\partial z} \right) \frac{G_{\text{eff}}}{2} + \underbrace{\frac{\omega r_{\text{eff}} \tilde{I}}{2Q}}_{\text{Beam Loading term!}}$$

Some features:

- **Paraxial** +**Quasi-static** approximation
- **Beam Loading term**: **Decelerating** gradient dependent on **Intensity**.
- Assumes **causality**!
- Matches [2] for the TW ultrarelativistic case

[4] J. Olivares Herrador, D. Esperante Pereira, N. Fuster, B. Gimeno, and A. Latina, “Beam Loading Simulation for Relativistic and Ultrarelativistic Beams in the Tracking Code RF-Track”, in Proc. LINAC’22, Liverpool, UK, Aug.-Sep. 2022, pp. 569–572. doi:10.18429/JACoW-LINAC2022-TUPORI13

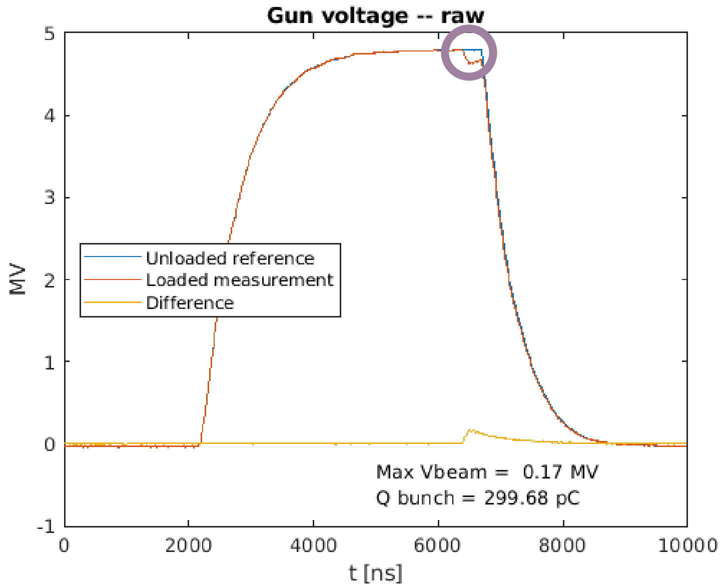
PART II: BL in SW photo-injectors

SW Beam Loading Model

- Photo-Injector = SW accelerating structure → v_g = 0 m/s.
- No power flow → Input power has to be considered

$$\frac{\partial G_{\text{eff}}}{\partial t} = -\frac{\omega}{Q}\frac{G_{\text{eff}}}{2} - \frac{\omega r_{\text{eff}} \tilde{I}}{2Q} + \frac{G_{\text{target}}\omega}{2Q}$$

Gradient reduction takes place while cavity is being fed



> Voltage along an RF-cycle for CLEAR SW photo-injector. Measurement.

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- v<c in the beginning ...
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- v<c in the beginning ...
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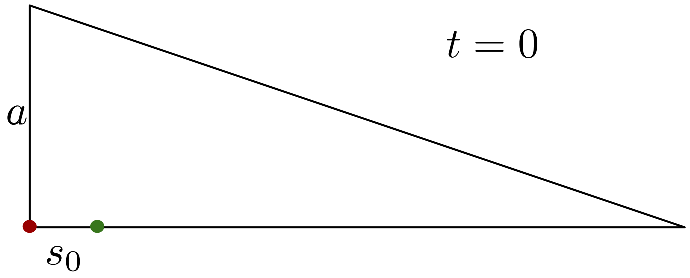
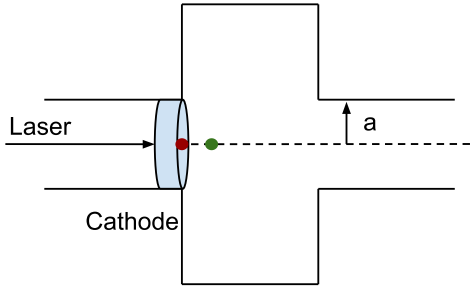
  - Causality?
    - Does the wake of the preceding particle affect the particle ahead?

To be addressed in each specific case

SW Beam Loading Model: Causality

- **Causality**: Does the wake of the preceding particle affect the particle ahead?





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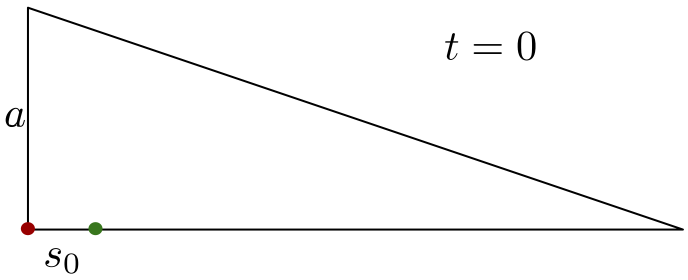
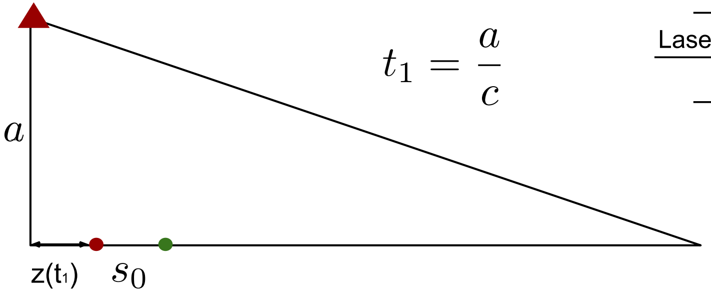
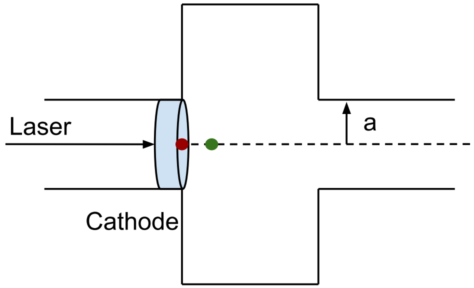


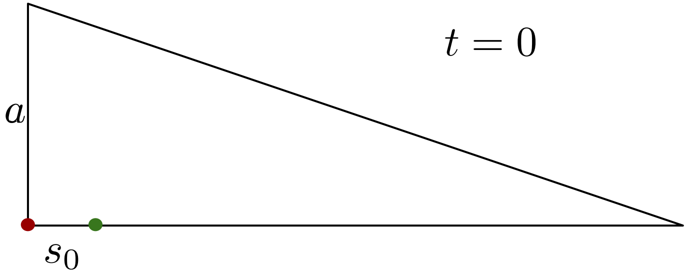




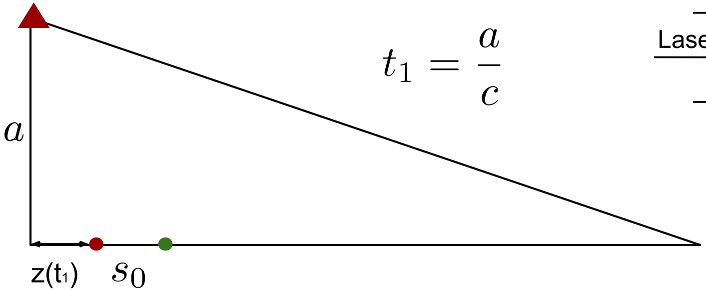
SW Beam Loading Model: Causality

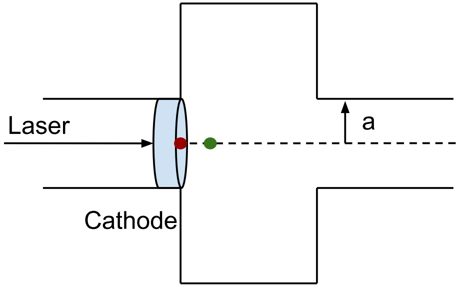
- **Causality**: Does the wake of the preceding particle affect the particle ahead?

$$t = 0$$

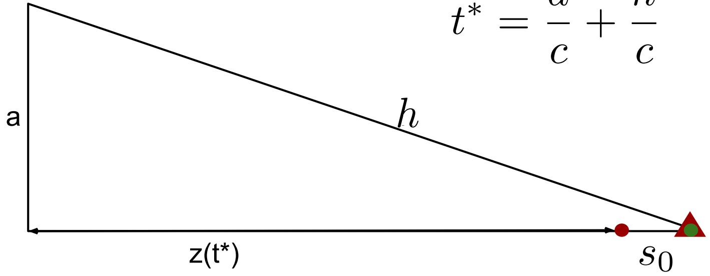


$$t_1 = \frac{a}{c}$$





$$t^* = \frac{a}{c} + \frac{h}{c}$$



$$\begin{cases} z(t^*) = \int_0^{t^*} \beta(t) c \, \mathrm{d}t \\ h = \sqrt{a^2 + (s_0 + z(t^*))^2} \end{cases}$$

$$\boxed{\exists t^* \in \mathbb{R} \text{ so that } t^* = \frac{a}{c} + \frac{1}{c}\sqrt{a^2 + (s_0 + z(t^*))^2}}$$

Catch-up condition

SW Beam Loading Model: Causality

- **Causality**: Does the wake of the preceding particle affect the particle ahead?
- CLEAR facility @ CERN

Magnitude	Units	Value
a	mm	10
$\langle G \rangle$	MV/m	30.2
σ_z	mm	$0.1 - 1.2$

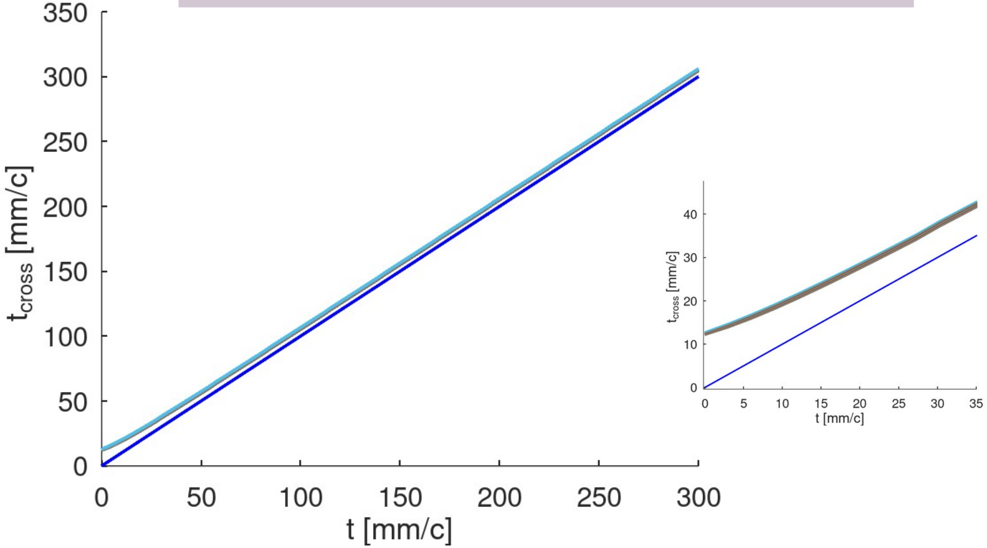
> CLEAR photo-injector information [5].

[5] https://clear.cern/content/beam-line-description

$$\exists t^* \in \mathbb{R} \text{ so that } t^* = \frac{a}{c} + \frac{1}{c}\sqrt{a^2 + (s_0 + z(t^*))^2}$$

Catch-up condition

No crossing → Causality not violated



> Catch-up condition verification plot with s0 ranging from 0 to 3σ.

PART III: BL Simulations in RF-Track

RF-Track

- About **RF-Track** **[6]**:
  - Beam tracking in field maps/analytic structures including **space-charge** effects, **wakefields**, …
  - Multiple species (arbitrary q and m)
  - **Parallel C++**, interface with user via **Octave** or **Python**

- **Beam Loading** in RF-Track:
  - Self-consistent module
  - Additional decelerating kick (F_{BL})
    - Attached to Drift spaces, Analytic TW & SW structures, field maps

[6] A. Latina. *RF-Track Reference Manual*. CERN, Geneva, Switzerland, June 2020 DOI: 10.5281/zenodo.3887085

BL in RF-Track

- Based on numerical resolution of $\frac{\partial G_{\text{eff}}}{\partial t} = -\frac{\omega}{Q}\frac{G_{\text{eff}}}{2} - \frac{\omega r_{\text{eff}}\tilde{I}}{2Q} + \frac{G_{\text{target}}\omega}{2Q}$
  - Finite difference method

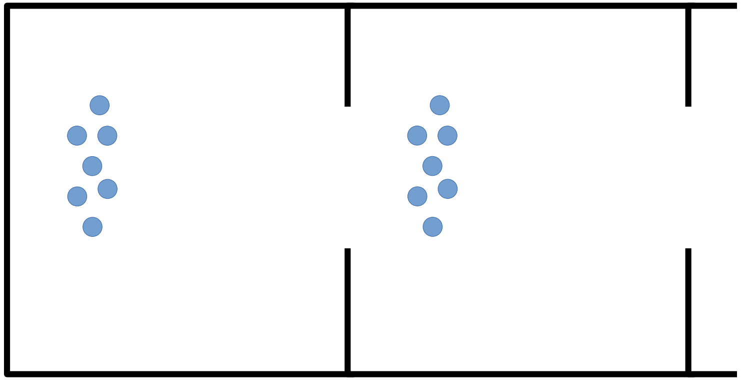
$$t \to \{t_m\}_{m=0}^{M-1} \quad z \to \{z_n\}_{n=0}^{N-1} \qquad G(z,t) \to G(z_n, t_m) := G(n,m) \qquad \tilde{I}(z,t) \to \tilde{I}(n,m)$$

$$\boxed{G(n,m+1) = \left(1 - \frac{\omega \Delta t}{2Q}\right) G(n,m) - C(n,m,\beta) + \frac{G_{\text{target}}\omega}{2Q}}$$

$$C(n,m,\beta) = \frac{\Delta t \omega r / Q \mathcal{T}(n,m,\beta)\tilde{I}(n,m,\beta)}{2}$$

BL in RF-Track

- **INPUT**: BEAM, Q, r/Q, $E_z(z,t=0)$, t_{inj}
- **Phase 1**: Preparation: G(z, t=0); Initialize m = 0
- **Phase 2**: Compute force while tracking



> Tracking simulation sketch.

$$\text{At } t_m = m\Delta t$$

$$\tilde{I}(z_n, t_m)$$

$$\text{Get } G(n, m+1)$$

$$\text{Define } \chi(n,m) = 1 - \frac{G(n,m+1)}{G(n,0)}$$

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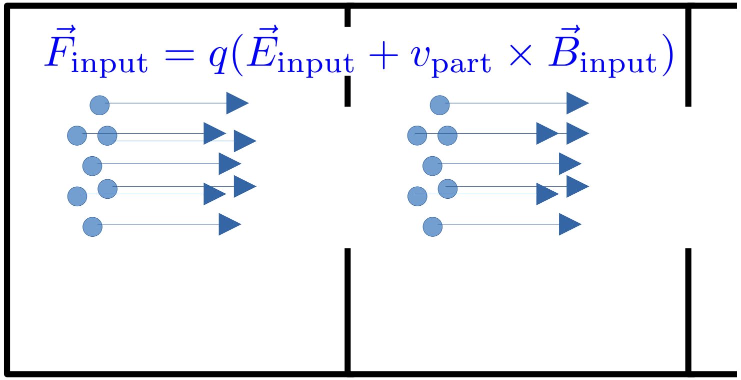

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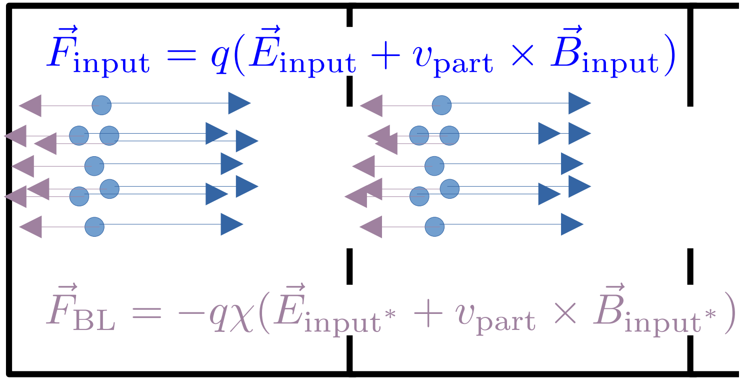

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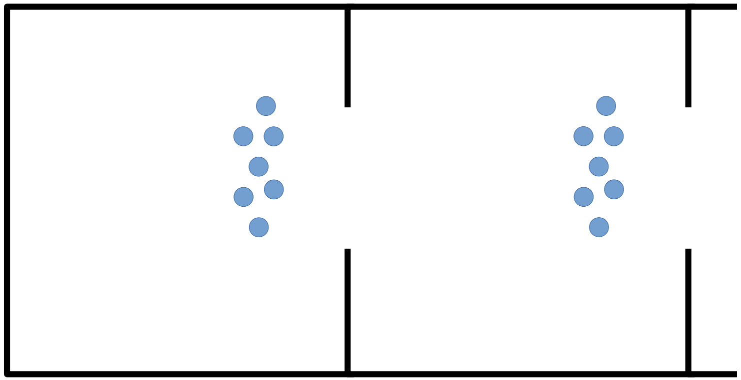


> Tracking simulation sketch.

$$\text{At } t_{m+1} = (m+1)\Delta t$$

$$\tilde{I}(z_n, t_{m+1})$$

$$\text{Get } G(n, m+2)$$

$$\text{Define } \chi(n, m+1) = 1 - \frac{G(n,m+2)}{G(n,0)}$$

Stop when all particles leave volume

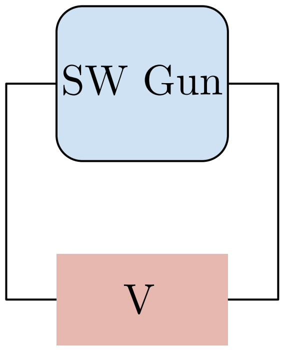
PART IV: Results

BL @ CLEAR photo-injector

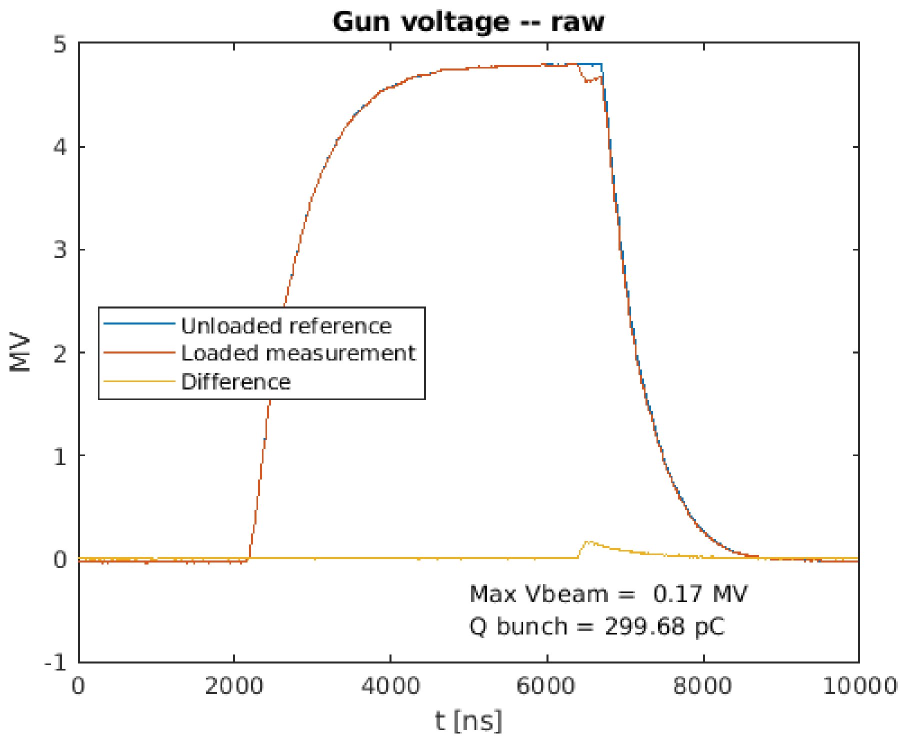
- Voltage measurements for gun-loop

Magnitude	Units	Value
f	GHz	2.997
ϕ_{ad}	rad	π
L_{total}	m	0.175
Q_0		15773
Q_l		5920
r/Q	Ω/m	3080
$E_z^{\max}$	MV/m	60.8

> CLEAR photo-injector information [5].



> Sketch of voltage measurement at CLEAR photo-injector.

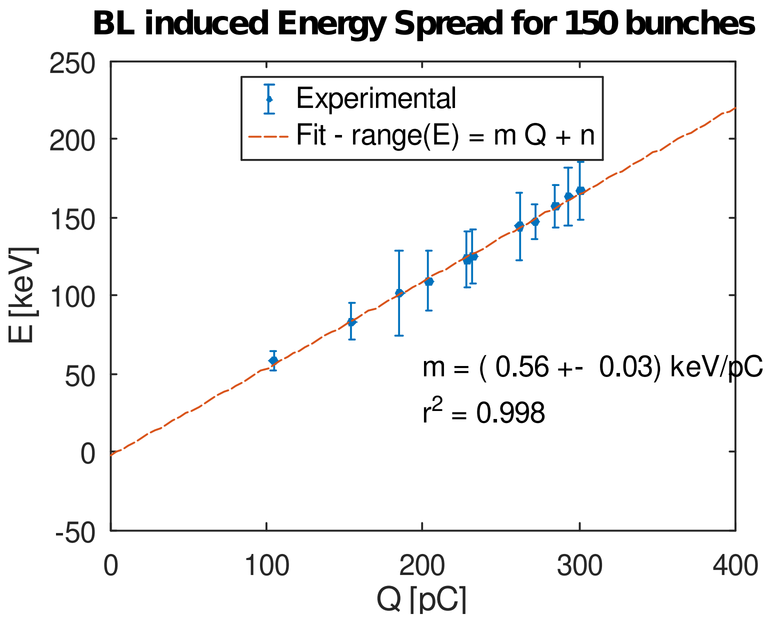


> Voltage along an RF-cycle for CLEAR SW photo-injector. Measurement.

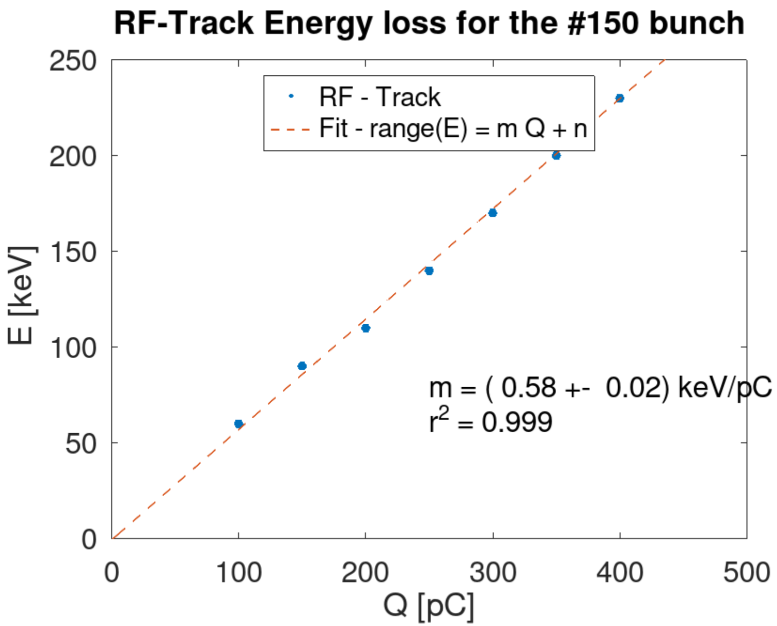
[5] https://clear.cern/content/beam-line-description

BL @ CLEAR photo-injector

- Train of 150 bunches with variable charge (Q_{bunch}) per bunch; $f_b = f_{RF}/2$



> Beam Loading Energy Spread induced in a train of 150 bunches as a function of charge



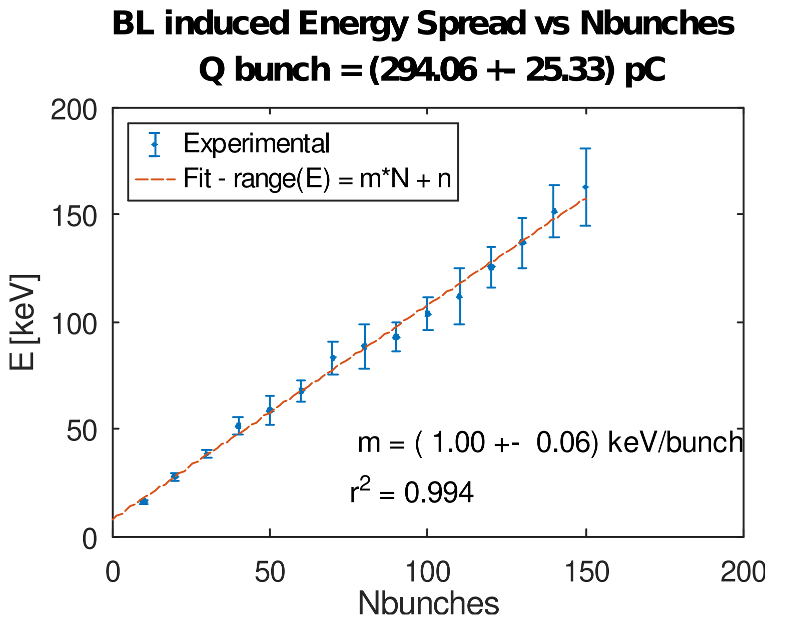
> Beam Loading Energy Spread induced in a train of 150 bunches as a function of charge (RF-Track, corrected attempt)

$$\delta_m = \frac{|m_{\text{exp}} - m_{\text{RF-Track}}|}{m_{\text{exp}}}$$

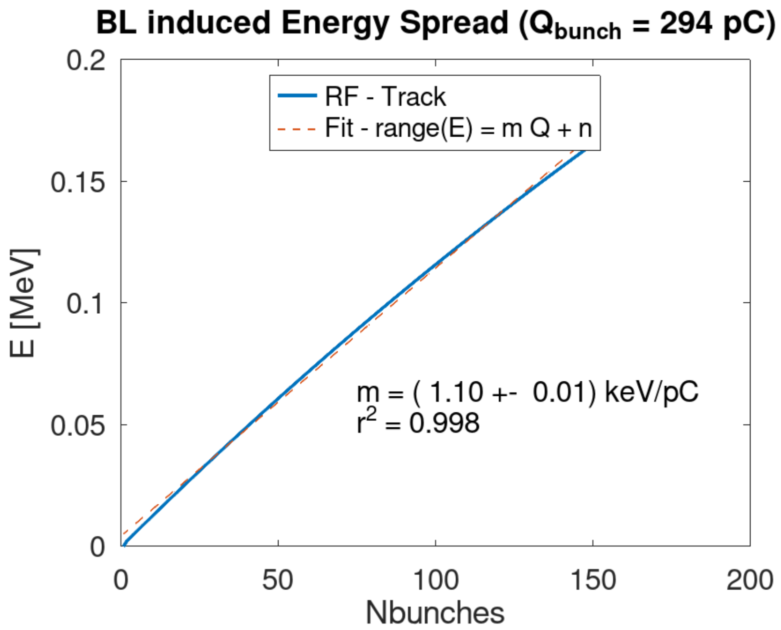
$$\delta_m = 3.5\%$$

BL @ CLEAR photo-injector

- Train of variable $N_{bunches}$ with fixed charged per bunch



> Beam Loading Energy Spread induced in a train with varying number of bunches and fixed charge per bunch.

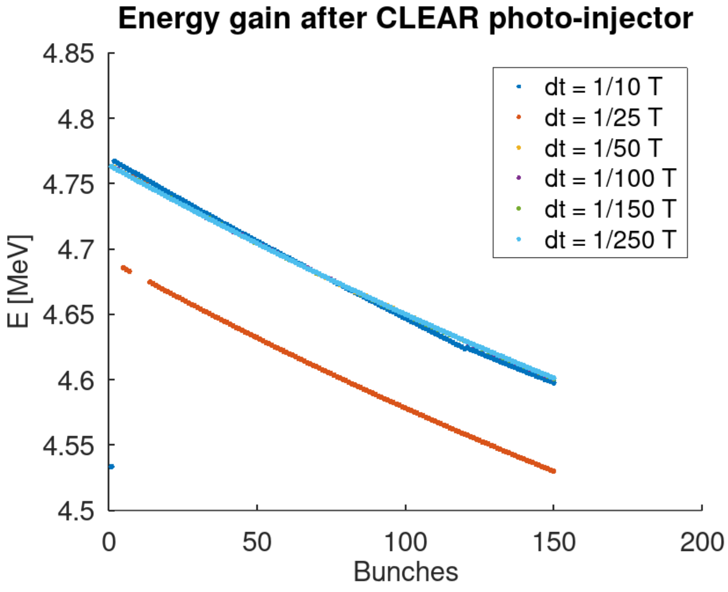


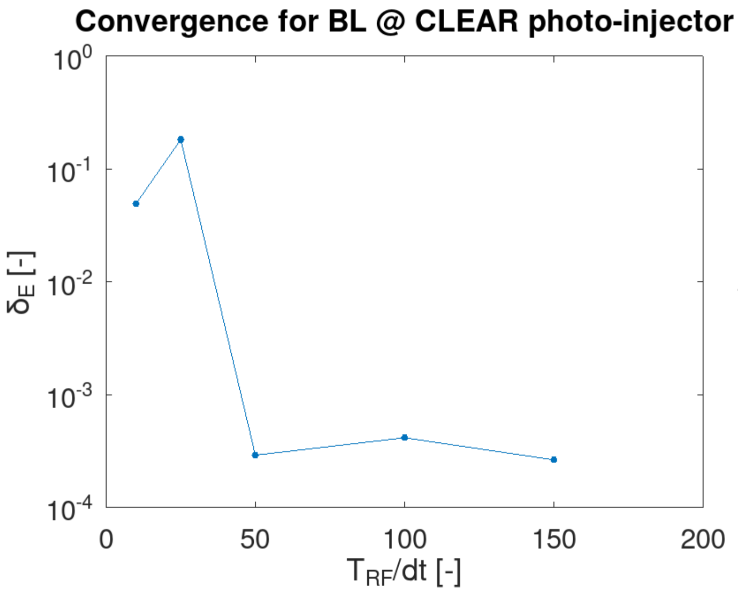
> Beam Loading Energy Spread induced in a train with varying number of bunches and fixed charge per bunch.

$$\delta_m = 10.0\%$$

BL in RF-Track. Convergence

- Proof of **convergence** under suitable ¢t choice





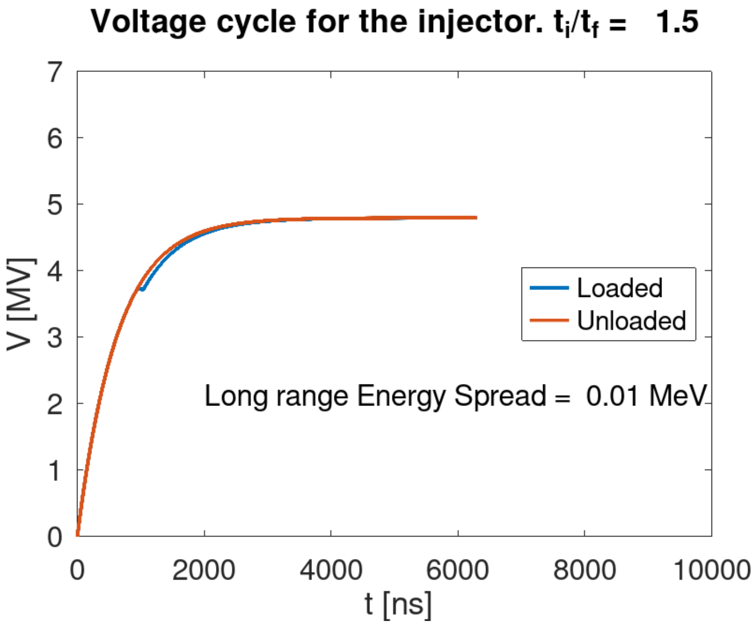
$$\delta_{E,n} = \sqrt{\sum \frac{(E_n - E_{1000})^2}{E_{1000}^2}}$$

> Energy gain of a train of 150 bunches with a charge per bunch of 300 pC.

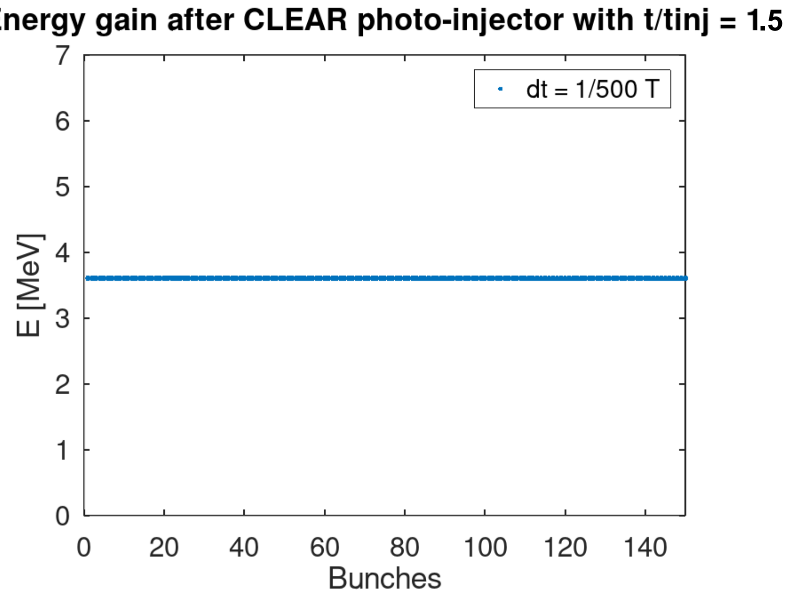
> Relative deviation of the train energy with respect to a highly accurate (E1000) energy distribution as a function of the dt/T fraction.

BL compensation

- BL can be compensated with **early injection** of the particles
  - **RF-Track** allows the simulation of this scenario



> Voltage along an RF-cycle for CLEAR SW photo-injector.



> Energy gain of a train of 150 bunches with a charge per bunch of 300 pC.

$$t_{\text{inj}} = \frac{2Q_l}{\omega}$$

Conclusions

- Beam Loading effect causes gradient reduction in **high-gradient & high-intensity** accelerators.
- The implementation of this model in **RF-Track** provides a user-friendly, flexible and powerful tool which shows:
  - Numerical **convergence & stability**

… and allows:

  - Simulation of **realistic** scenarios
    - Good **agreement with measurements** at the CLEAR photo-injector
  - Beam Loading **compensation**

OUTLOOK: Further development of the code: Interaction of BL with other collective-effects & Start-to-end simulations.

Acknowledgements

- **Supervision**, guidance and trust:
  - Andrea Latina (CERN, BE-ABP-LAF)

    also, RF-Track creator and main developer.
  - Nuria Fúster Martínez, Benito Gimeno, Daniel Esperante (UV – Spain, IFIC - CSIC)

- Useful **material & discussions**:
  - Alexej Grudiev (CERN, SY-RF-MKS)
  - Avni Aksoy (CERN, BE-ABP-LAF)
  - The CLEAR OP team

Thanks for your attention

LEDS2023

Longitudinal Electron beam Dynamics for coherent light Sources

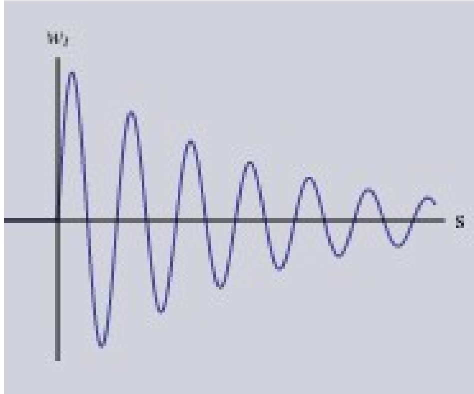
BACK UP SLIDES

Back up slide – Causality (General)

- **Shunt impedance** is defined as:

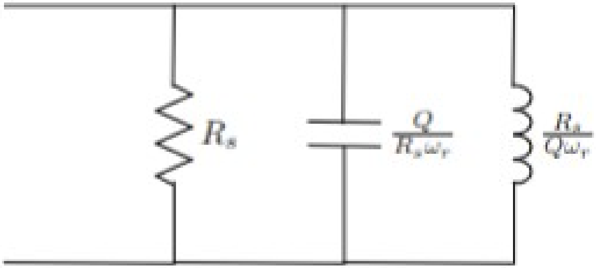
$$R_s = \frac{V_{\text{acc}}^2}{P_{\text{diss}}}$$

- Such definition comes from the LCR circuit analogy
  - Which comes from the fact of considering a **causal longitudinal wakefield**



Causal longitudinal wake

$$\Longrightarrow Z_l \simeq \frac{R}{1 + jQ\left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r}\right)} \Longrightarrow$$



Accelerator behaves as parallel LCR circuit in resistive mode

GUN: (Ez, φ) Calibration

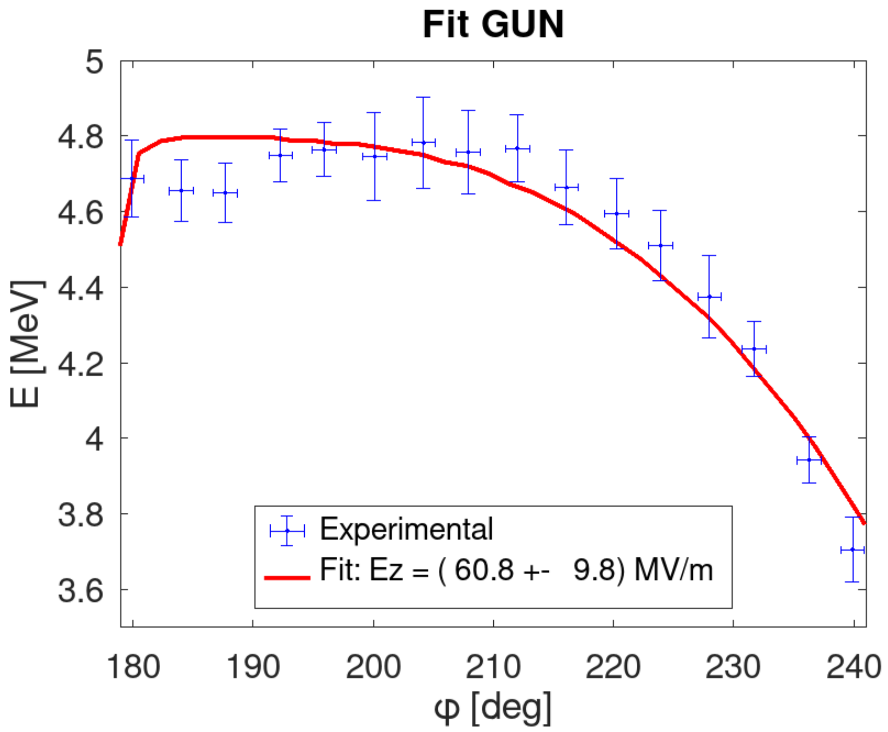
With Avni Aksoy

- For a 50 pC bunch:
  - Collect (E_k, φ_k) measurements
  - Fit then to target function F(E_z, φ)
    - F: RF-Track calculation of E after gun.

Magnitude	Units	Value
E_z^{max}	MV/m	60.8 ± 9.8
r^2		0.94

> Results of the minimum square fitting with a test function computed with RF-Track.

Still cathode influence to be studied!



> Energy gain after the gun as a function of the phase. In red, RF-Track prediction. In blue, experimental results

GUN: Beam Loading Measurements

- 2) Divergent slope
  - Looking again at the BL equation …

$$-\frac{\partial G}{\partial t} = \frac{\omega}{Q}\frac{G}{2} + \frac{\omega r_{\text{eff}} I}{2Q} + \frac{G_{\text{init}}\omega}{2Q}$$

  … the slope of the plot E vs Q depends on **r/Q and Q**

  - From design report: r/Q = ; Q0 = 14530;
  - However, we learn that the Q governing the dynamics is

$$Q = (598 \pm 8) \cdot 10$$

  - This is the loaded quality factor!

$$Q_l = \frac{Q_0}{1+\beta} = (598 \pm 8) \cdot 10 \implies \beta = 1.5$$