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# Studies on transient beam loading generated by the superconductive harmonic cavity

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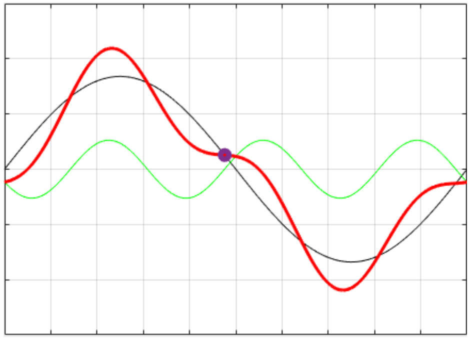
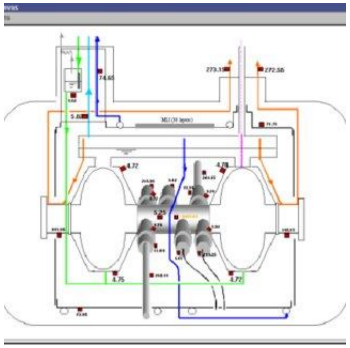
# Outline

- Transient beam loading due to a passive harmonic cavity and uneven filling pattern
- Simulator based on analytic frequency-domain model
- Elettra 2.0 studies
- Conclusions
- (Optional: measurements and 3HC characterization in the Elettra storage ring)

# 3HC Impact on Longitudinal Beam Dynamics





3HC

superconductive passive

third harmonic cavity

- Bunch lengthening ☺
  - Lifetime increase
  - Landau damping
- 3HC+dark gap: Transient Beam Loading ☹
  - Variation of beam characteristics along the bunch train: stable phase, synchrotron tune, charge profile, bunch length, bunch lifetime, …

For Elettra 2.0 we need an estimation of the transient beam loading due to the third harmonic cavity to evaluate impacts on MBF systems (longitudinal modulator working at the 4$^{th}$ RF harmonics), crab cavities, hybrid mode, …

# Transient Beam Loading

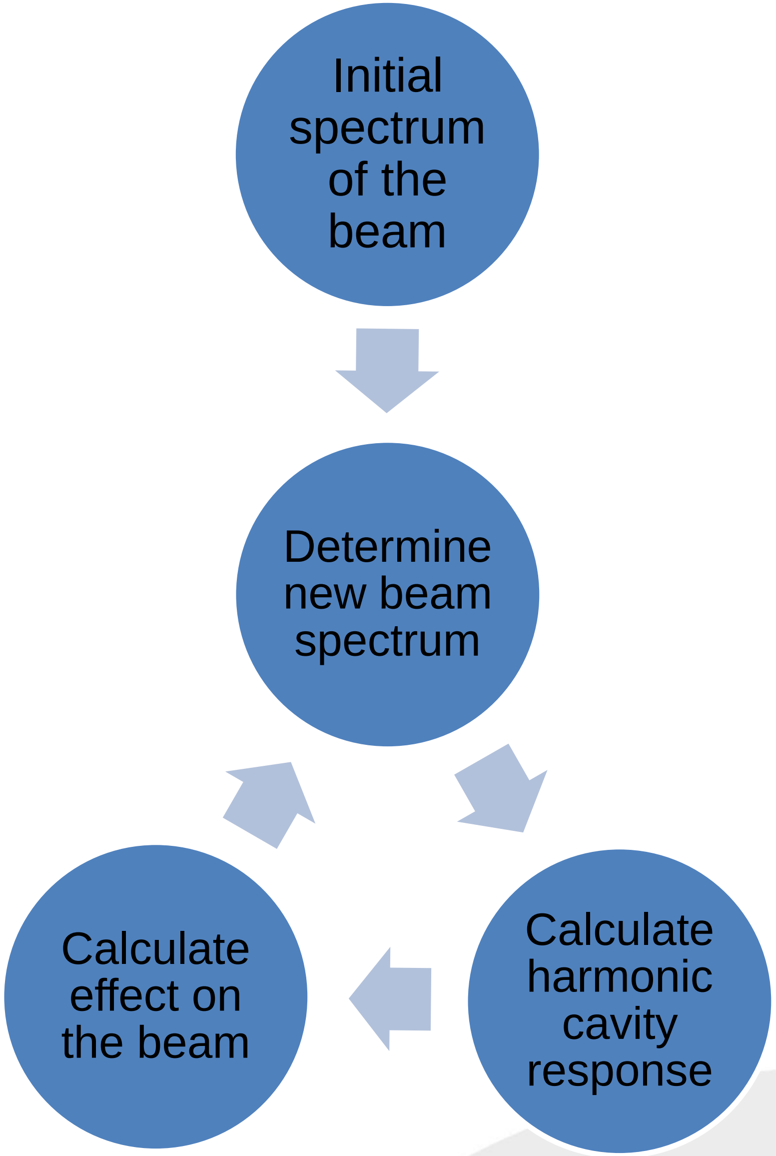
- Transient beam loading is produced by RF cavities in the presence of a dark gap in the filling pattern
- Generally speaking, the beam loading is the voltage induced by the beam in a cavity at the frequency of the bunches; in the fraction of one machine turn where the buckets are not filled, the voltage in the cavity freely oscillates at the cavity resonant frequency
- This transient generates a periodic modulation of the cavity voltage, with period equal to the revolution time, resulting in different total voltage seen by the bunches depending on their longitudinal position
- As a result, the bunch characteristics, such as synchronous phase, synchrotron frequency, charge profile and lifetime change along the bunch train

# 3HC Model and Longitudinal Beam Dynamics

Simulator developed in MATLAB

Analytical frequency-domain model of the beam and harmonic cavity

- The beam is represented by a periodic ($\omega_r$) signal made of pulses at $\omega_{RF}$ modulated in amplitude and phase; has a given spectrum
- The harmonic cavity is a dynamic system with an INPUT (beam) and an OUTPUT (harmonic voltage)
- The harmonic gap voltage modifies the bunch length and position (long. displacement of the bunches with respect to synchronous phase)
- Iterate process until a stable, consistent solution is found: equilibrium bunch density. Normally a few iterations are sufficient to converge




**Simulator block diagram**

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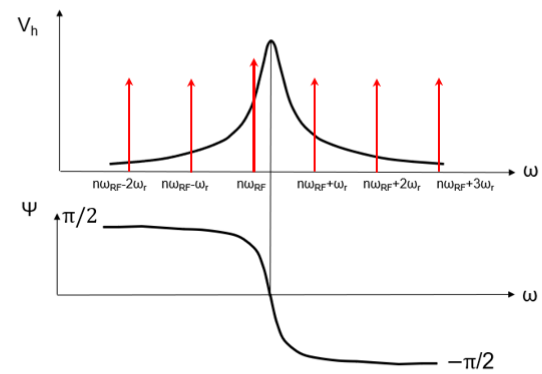
# Analytical equations of the Model

The beam can be modelled by the "complex beam current", an array of complex values each representing one of the bunches, where the module of each value is the bunch current and the phase is the bunch synchronous phase

1) $$\bar{F}(i) = \frac{\mathcal{F}(\Psi_i)(3\omega_{RF})}{\mathcal{F}(\Psi_i)(0)} = \frac{\int_{-\pi}^{\pi} \Psi_i\,(\varphi)\, e^{-j3\varphi} d\varphi}{\int_{-\pi}^{\pi} \Psi_i\,(\varphi)\, d\varphi}$$ Complex Form Factor (=1 initial conditions)

2) $$\bar{I}(i) = I_b(i)\;\bar{F}(i)$$ Complex Beam Current

3) $$|H_{3HC}| = \frac{R_s}{Q}\frac{\omega_{3HC}}{\omega_{3HC}-\omega}\sin \operatorname{atan}\left(\frac{2Q(\omega_{3HC}-\omega)}{\omega_{3HC}}\right)$$
$$arg(H_{3HC}) = \operatorname{atan}\left(\frac{2Q(\omega_{3HC}-\omega)}{\omega_{3HC}}\right)$$ Harmonic cavity transfer function




4) $$V_{3HC}(\omega) = \mathcal{F}(\bar{I})(\omega)\,H_{3HC}(\omega)$$ Harmonic Cavity Voltage

5) $$V_{tot}(\varphi) = V_{RF}\sin(\varphi + \varphi_s) + V_{3HC}\sin(3\varphi + \varphi_{3HC})$$ Total voltage seen by the bunches

6) $$U(\varphi) = -\frac{c\,\alpha}{E C \omega_{RF}}\int_0^{\varphi}(eV_{tot}(\varphi') - U_0)d\varphi'$$ RF potential

7) $$\Psi(\varphi) = \frac{e^{-\frac{U(\varphi)}{\alpha^2\sigma_e^2}}}{\int_{-\pi}^{\pi} e^{-\frac{U(\varphi)}{\alpha^2\sigma_e^2}} d\varphi}$$ Charge distribution

- The equilibrium bunch density is obtained by iterating this process until a consistent stable solution is found; convergence is reached after ~200 iterations and a few seconds
- From the equilibrium bunch charge distribution we can calculate the bunch characteristics (stable phase, bunch length, Touschek lifetime, etc.); the synchrotron frequency is determined analytically from the gradient of the total voltage seen by each bunch

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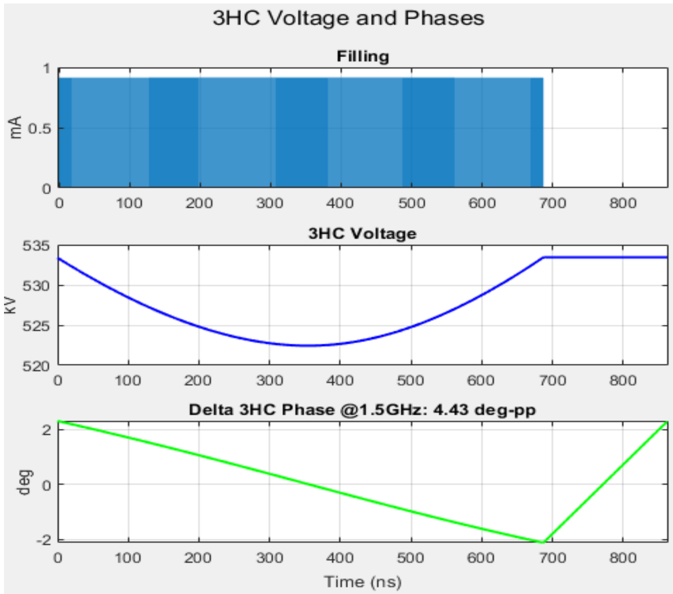
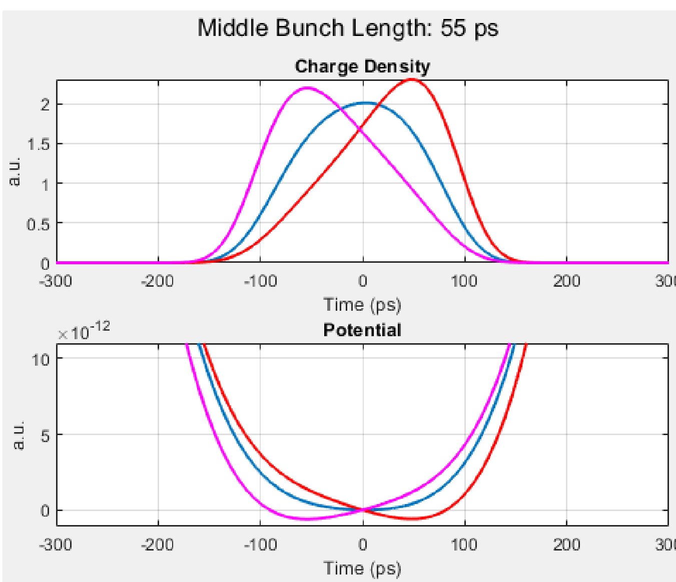
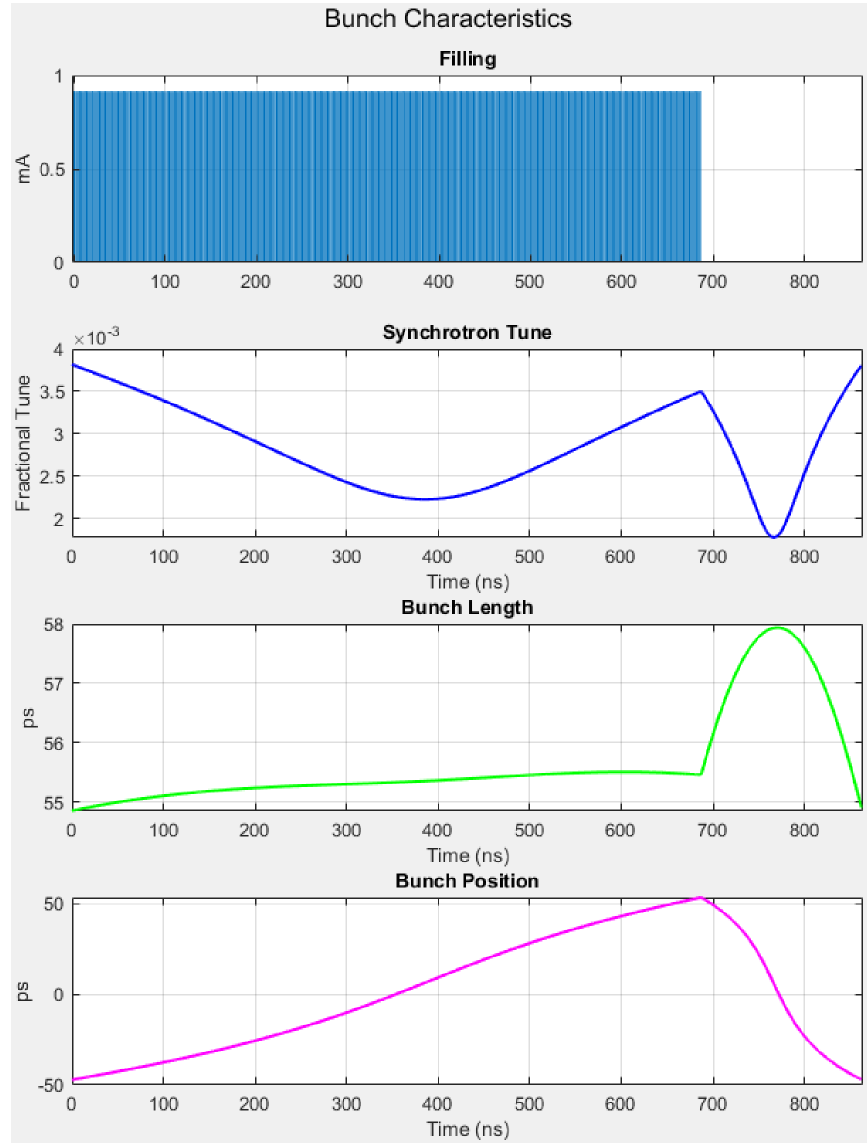
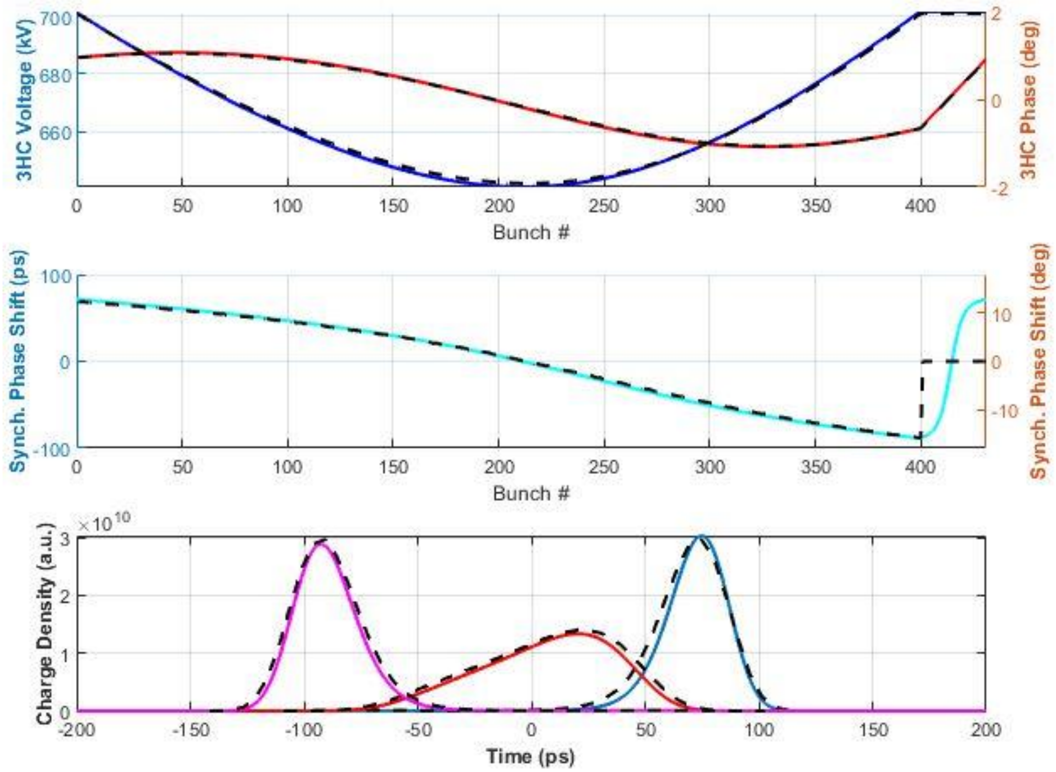
# Results: harmonic cavity response and bunch characteristics



# Simulations compared with *mbtrack2*




Comparison of results obtained from the Matlab simulator (solid lines) and *mbtrack2* (dashed lines): 3HC voltage/phase and synchronous phase shift along the bunch train, and charge profile of the first/middle/last bunch (blue/red/magenta)

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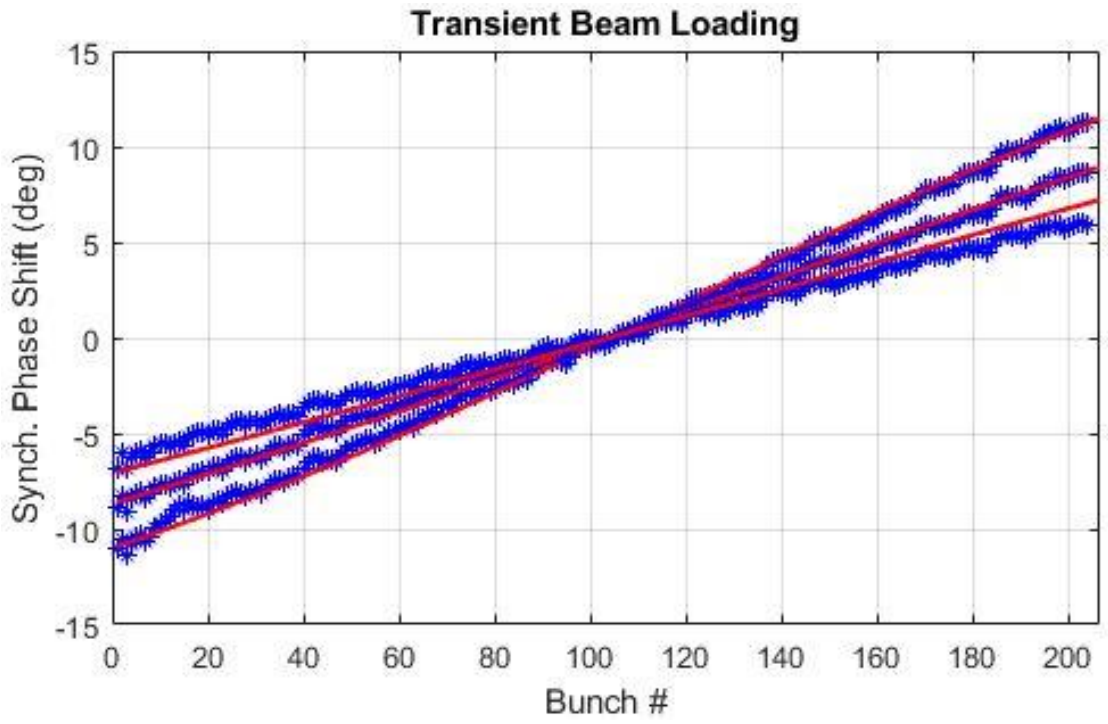
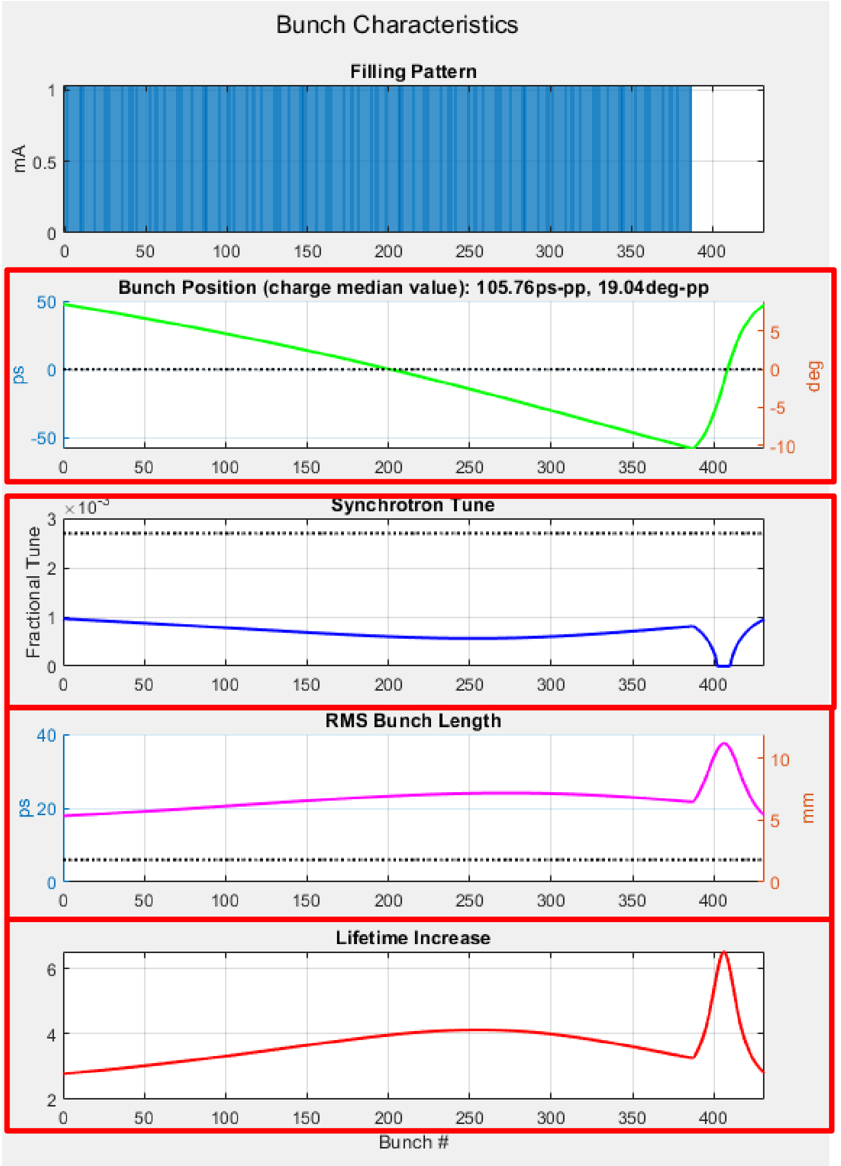
# Simulations compared with real Elettra machine




Synchronous phase shift along the bunch train; simulation (red) and experimental (blue) data, with E = 2.0 GeV, Ib = ~200 mA, 50% filling pattern and three different detuning values 45, 60 and 70 kHz

# Elettra 2.0 studies




Concerns about the impact of transient beam loading on Elettra 2.0 beam characteristics

Stable phase variation, concern for: synchronous detectors, MBF, Crab, …

Low Synchrotron Frequency; concern for Mode Coupling Instability (variation of frequency in the train could help?)

Ununiform bunch lengthening: concern for the single bunch in the gap (Crab cavities)

Ununiform lifetime increase: can spoil the rectangular filling pattern

# Transient beam loading effects vs. 3HC detuning

Electron beam

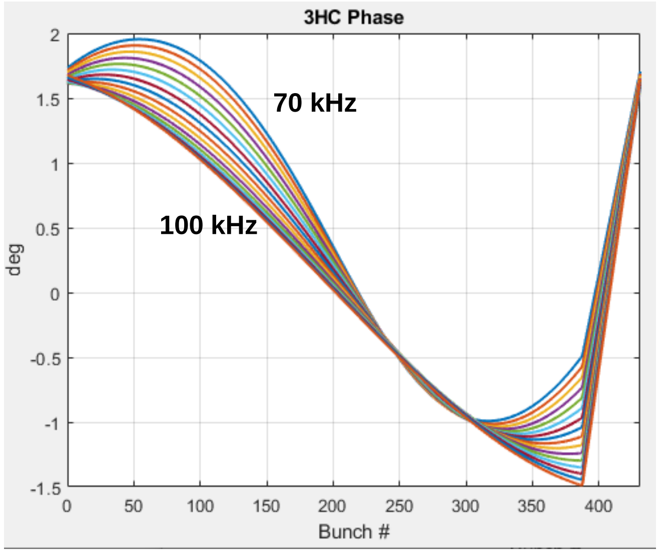
- Energy: 2.4 GeV
- Total current: 400 mA
- filling pattern: 90%
- 3HC and main RF cavity included
- 3HC detuning from 70 to 100 kHz

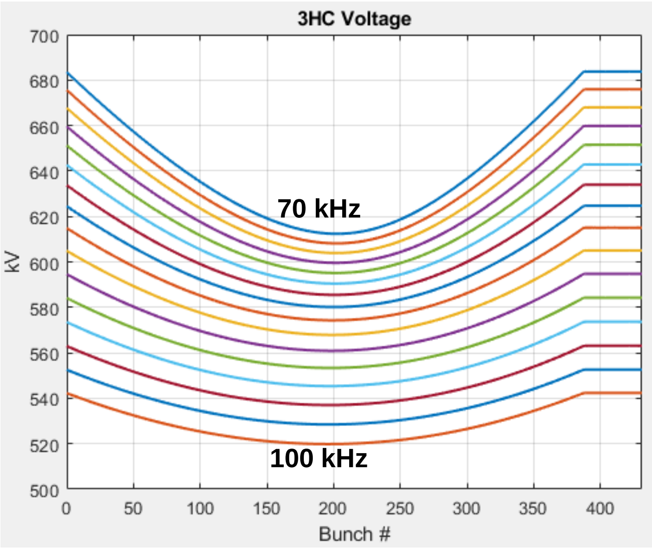
RF

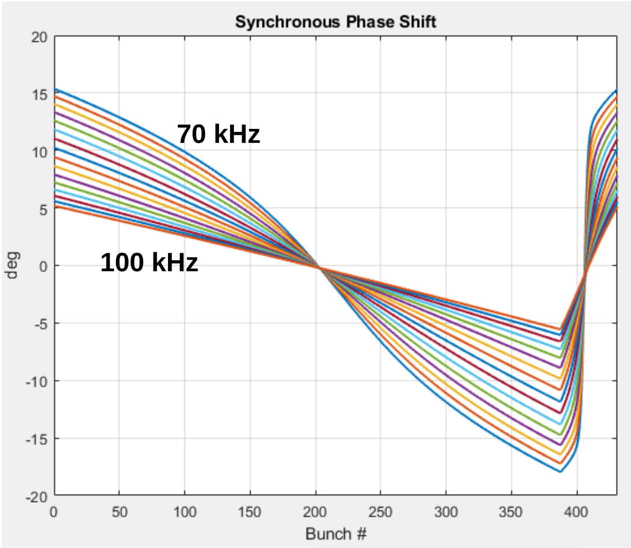
- Frequency: 500 MHz
- Total voltage: 2 MV
- Detuning: 1 kHz
- Q: 40000
- $R_S$: 3.3 MΩ

3HC

- Q: $2 \cdot 10^8$
- $R_S/Q$: 88 Ω



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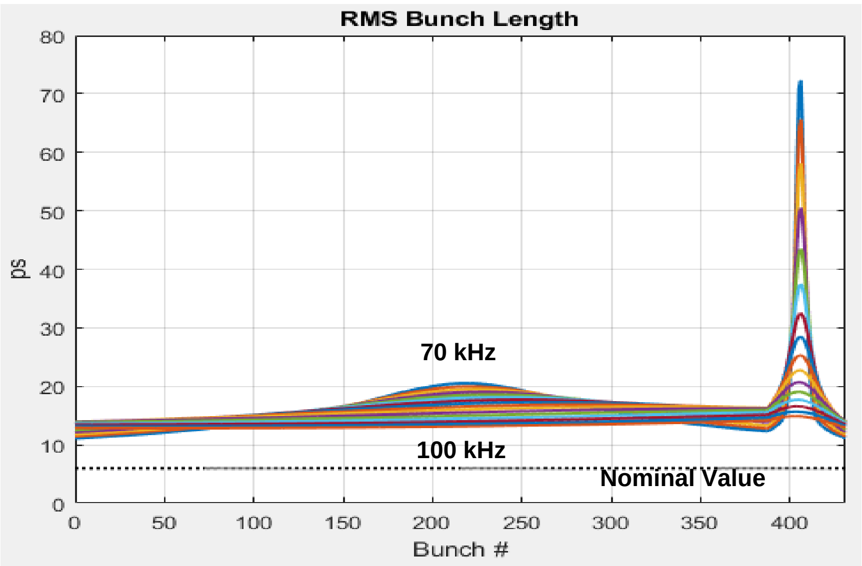
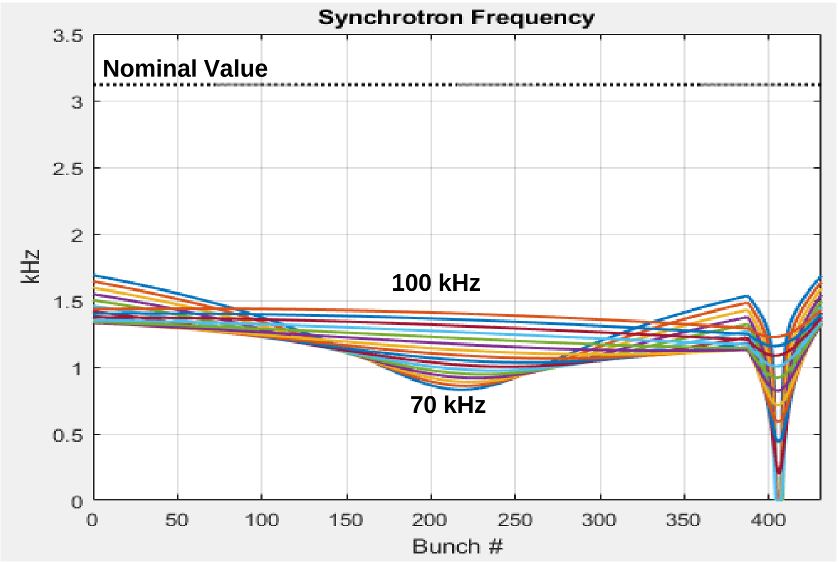
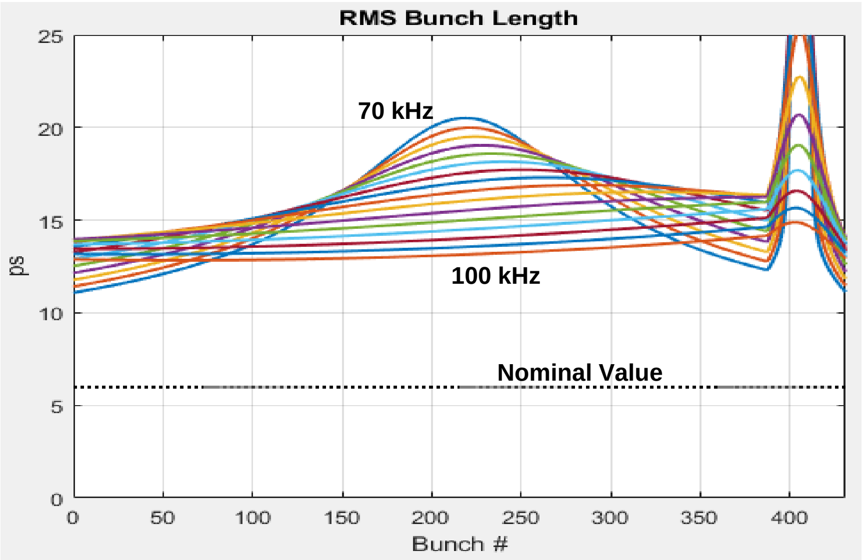
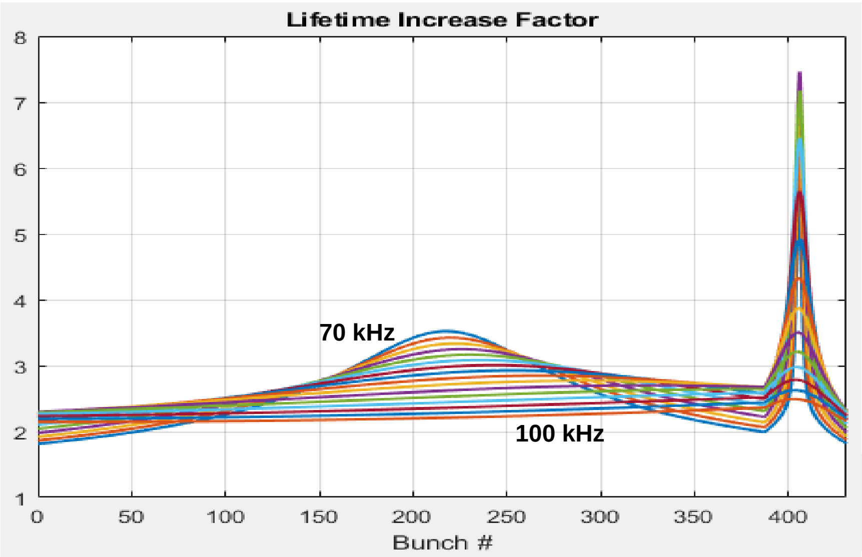
# Transient beam loading effects vs. 3HC detuning

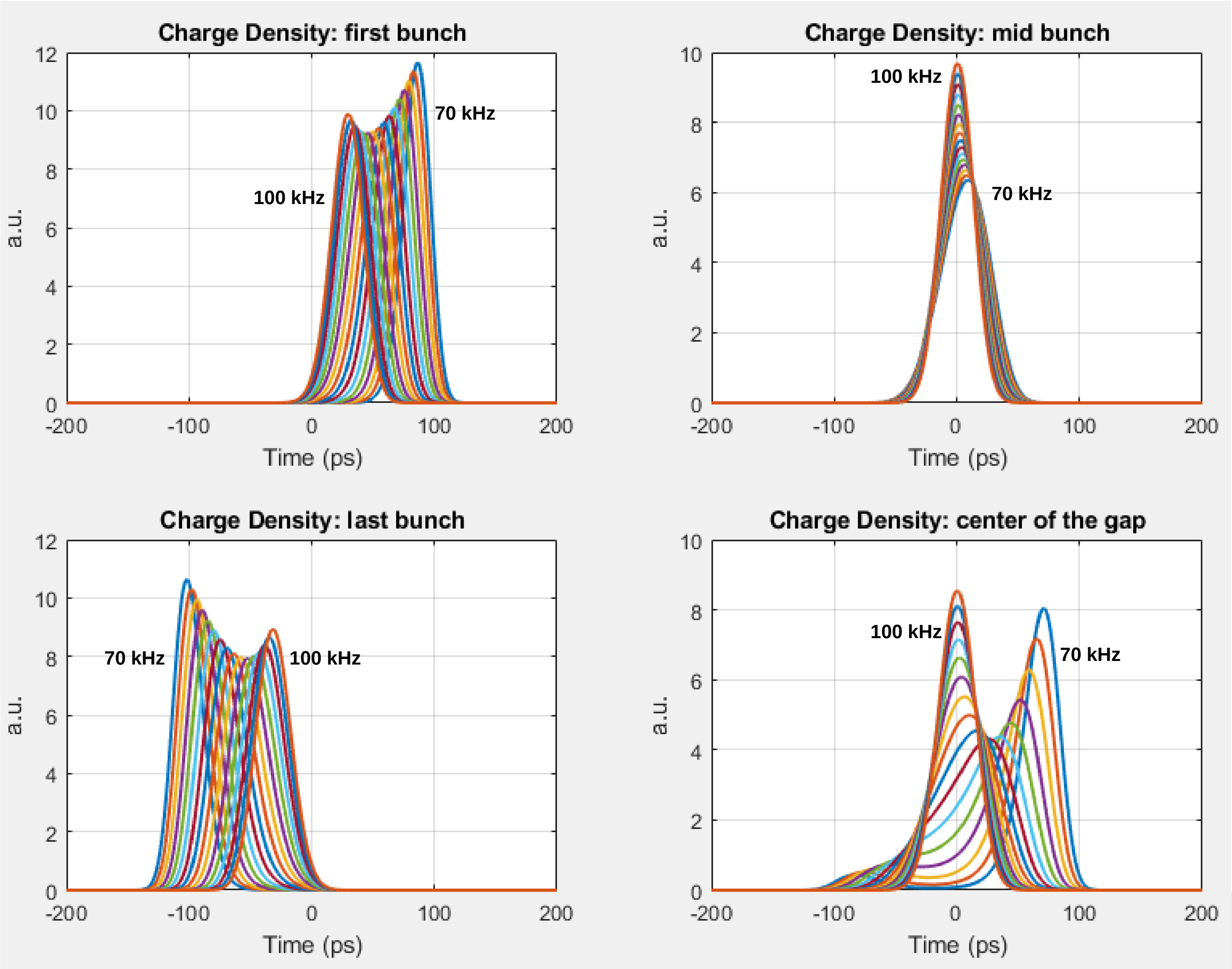


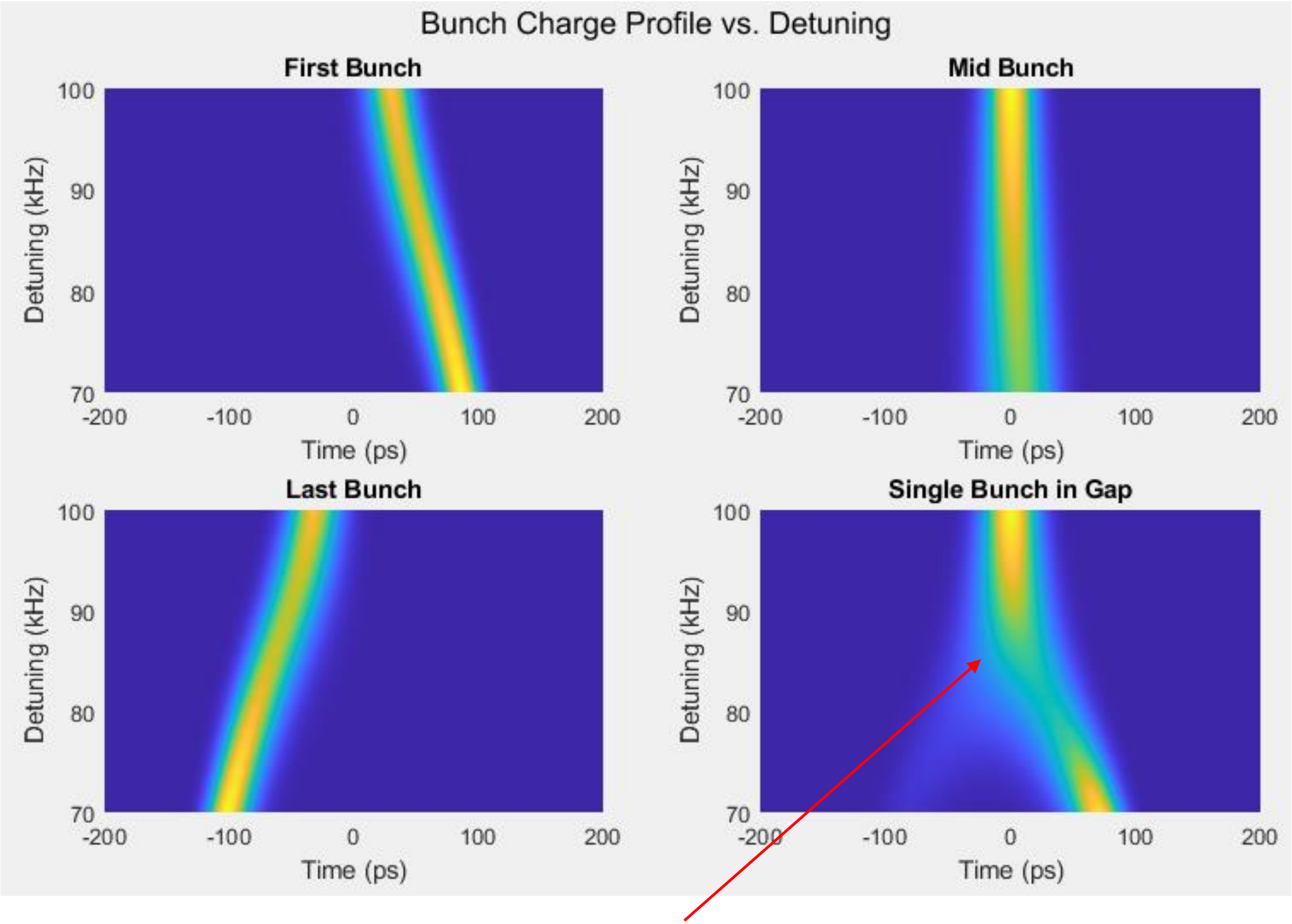



# Bunch charge profile vs. 3HC detuning



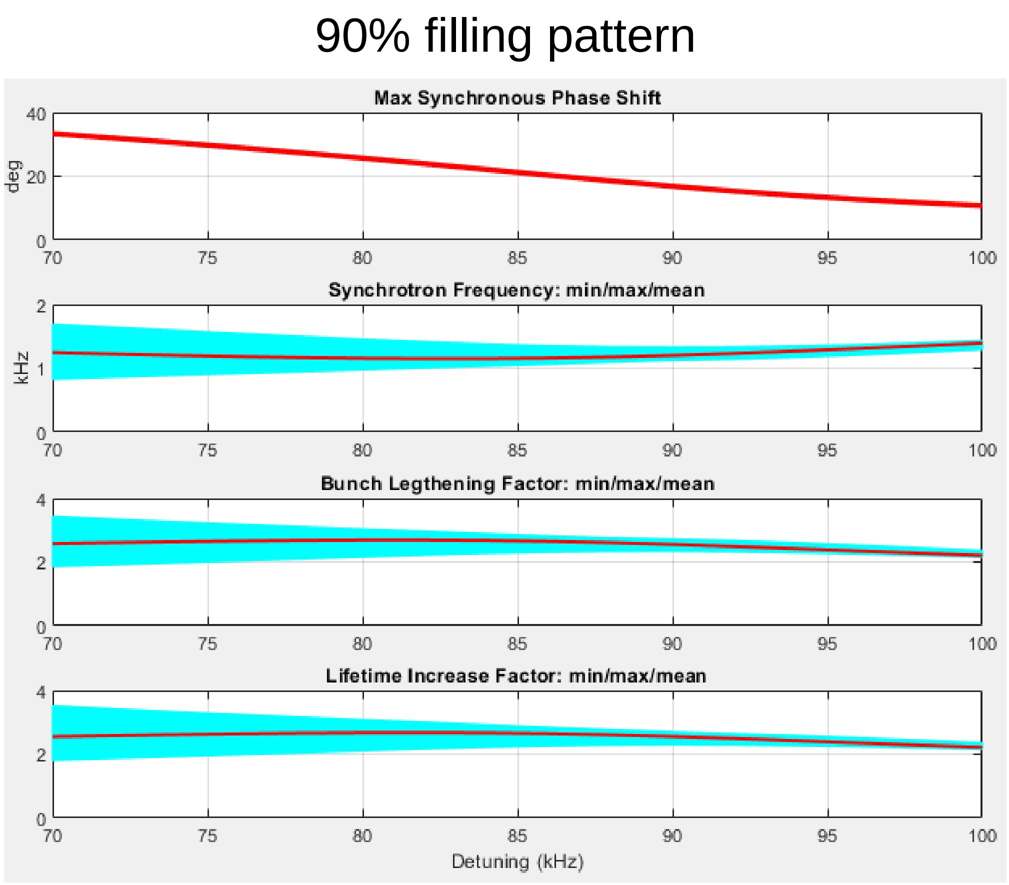
# Bunch charge profile vs. 3HC detuning

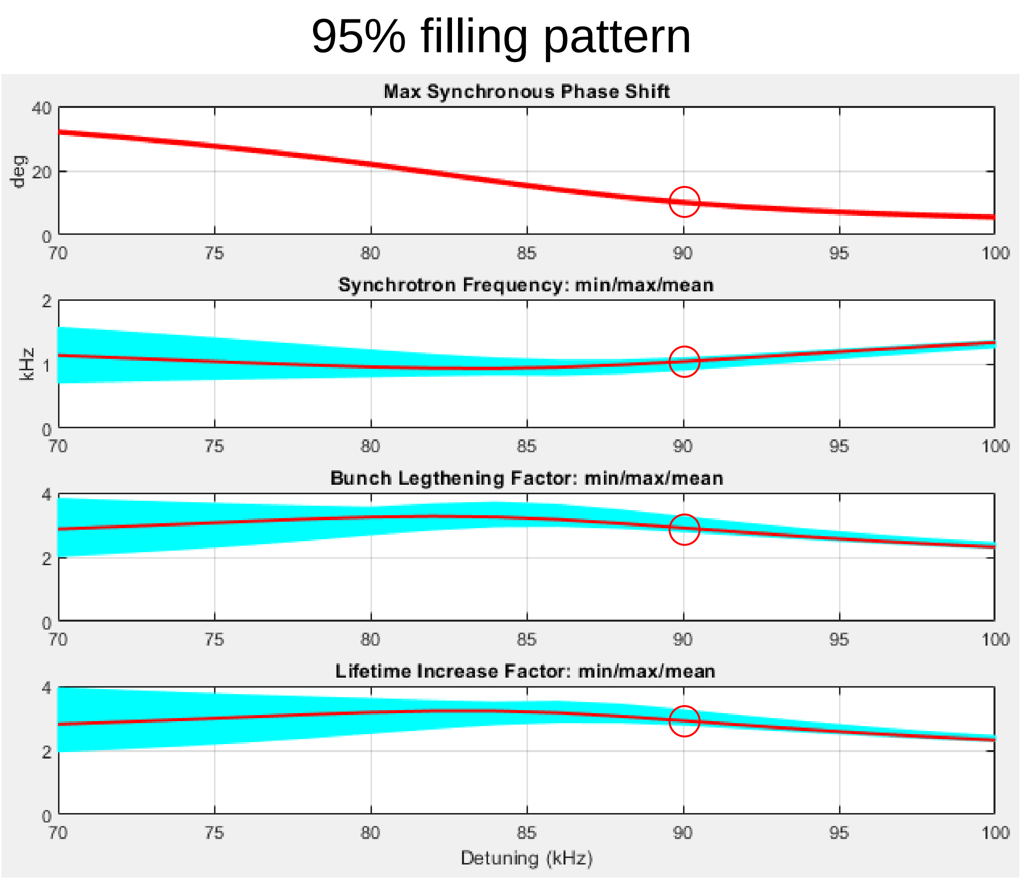



The single bunch in the gap is the most affected by transient beam loading

Decreasing the detuning it starts splitting into two bunches

# Bunch characteristics vs. harmonic cavity detuning




In order to have a good and uniform bunch lengthening and a reasonable synchronous phase shift along the bunch train, a good compromise is:
**detuning = 90 kHz at 95% filling pattern**

- Max synchronous phase shift: 10º
- Synchrotron frequency: ~1 kHz
- Bunch lengthening factor: ~ 3
- Touschek lifetime increase factor: ~ 3

# Conclusions

- Simulator based on an analytical frequency-domain model of the beam and third harmonic cavity (3HC), equilibrium bunch density distribution is obtained via an iterative process: convergence is obtained in a few seconds
- Results in good agreement with *mbtrack2* and Elettra experimental data
- Transient beam loading due to the main RF accelerating cavities included in the simulator
- The simulator is very efficient but is not suitable to study instabilities (Robinson, PTBL, …); *elegant* or *mbtrack2* have to be used
- Simulations of Elettra 2.0 have been carried out: a reasonable compromise has been found with limited negative impact of 3HC on the beam

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# References


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# Thank you!


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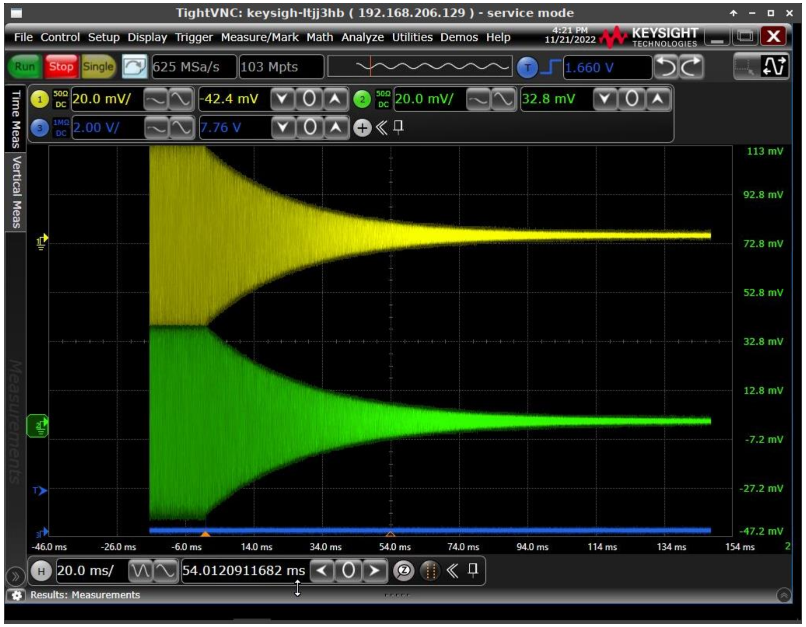
# Elettra Machine and 3HC Characterization: 3HC Quality Factor

The decay time of the 3HC voltage after a commanded beam dump has been measured by acquiring with an oscilloscope the signal taken from both cavity pickups. The acquired data have been digitally I/Q demodulated and the amplitude decay fitted with exponential curves, eventually providing the time constant and thus the Q factor.




$$Q = \frac{2\tau}{\omega_0}$$

| Cell# | Current | Detuning | τ | Q |
|---|---|---|---|---|
| | (mA) | kHz | ms | x $10^8$ |
| 1 | 120 | 40 | 28.3 | 1.33 |
| 1 | 120 | 50 | 28.3 | 1.33 |
| 1 | 120 | 60 | 28.3 | 1.33 |
| 1 | 120 | 70 | 28.3 | 1.33 |
| 1 | 120 | 80 | 28.3 | 1.33 |
| 1 | 300 | 70 | 27.9 | 1.31 |
| 1 | 300 | 90 | 28.4 | 1.34 |
| 2 | 120 | 40 | 30.7 | 1.45 |
| 2 | 120 | 50 | 30.7 | 1.45 |
| 2 | 120 | 60 | 30.7 | 1.45 |
| 2 | 120 | 70 | 30.7 | 1.45 |
| 2 | 120 | 80 | 30.7 | 1.45 |
| 2 | 300 | 70 | 30.4 | 1.43 |
| 2 | 300 | 90 | 30.2 | 1.42 |

# Elettra Machine and 3HC Characterization: 3HC Rs/Q

By measuring the synchrotron frequency of the central bunch in the bunch train, which is almost unaffected by the phase shift due to beam loading, we can first calculate the 3HC voltage and then Rs/Q. Preliminary measurements: calculated Rs/Q value 80 Ω, nominal value 88 Ω.

$$V_{3HC} = I_b \left(\frac{R_s}{Q}\right) \frac{\omega_{3HC}}{\delta\omega}$$

$$\left.\begin{aligned} V_{tot}(\varphi) &= V_{RF}\sin(\varphi + \varphi_s) + V_{3HC}\sin(\varphi_{3HC} - 3\varphi) \\ \frac{dV_{tot}}{d\varphi} &= V_{RF}\cos(\varphi + \varphi_s) - 3\,V_{3HC}\cos(\varphi_{3HC} - 3\varphi) \\ \varphi_s &= \operatorname{asin}\left(\frac{U_0}{eV_{RF}}\right) \end{aligned}\right\}$$

For the central bunch $\varphi_{3HC} = \varphi = 0$ ⇨ $\frac{dV_{tot}}{d\varphi} = V_{RF}\cos\varphi_s - 3\,V_{3HC}$

Synchotron Tune

$$Q_s = \sqrt{\frac{\alpha h \frac{dV_{tot}}{d\varphi}}{2\pi E_0}}$$

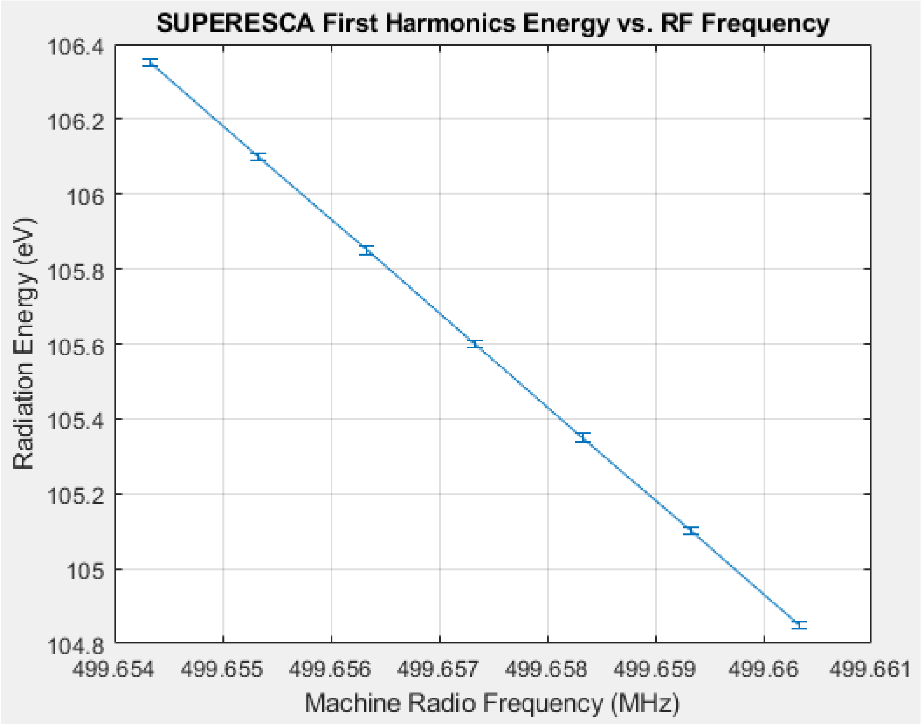
$$Q_s = \sqrt{\frac{\alpha h\left(V_{RF}\cos\operatorname{asin}\left(\frac{U_0}{eV_{RF}}\right) - 3V_{3HC}\right)}{2\pi E_0}}$$

$$V_{3HC} = \frac{V_{RF}\cos\operatorname{asin}\left(\frac{U_0}{qV_{RF}}\right) - \frac{2\pi E_0 Q_s^2}{h\alpha}}{3}$$

$$\frac{R_s}{Q} = \frac{V_{3HC}}{I_b}\,\frac{\delta\omega}{\omega_{3HC}}$$

# Elettra Machine and 3HC Characterization: Momentum compaction factor α

The momentum compaction factor α can be calculated with reasonable accuracy through measurements of the photon energy on a beamline at different electron energies.




$$\alpha = -2\frac{\frac{\Delta f_{RF}}{f_{RF}}}{\frac{\Delta E_p}{E_p}}$$

**$\alpha$ = 1.70 x $10^{-3}$ ± 1%**

(nominal value = 1.6 x $10^{-3}$)