

# Synchrotron radiation: Introduction to Synchrotron radiation and FEL

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Synchrotron radiation  
or  
synchrotron **light**  
is  
electromagnetic radiation

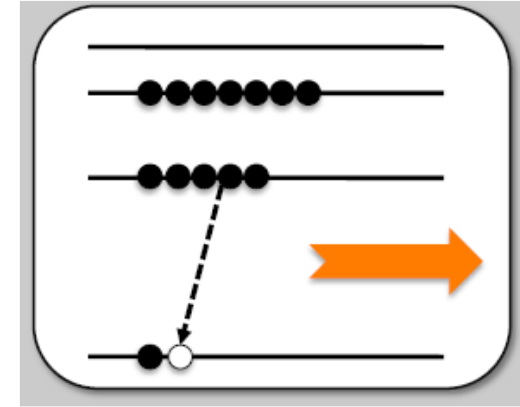


Elettra Sincrotrone Trieste

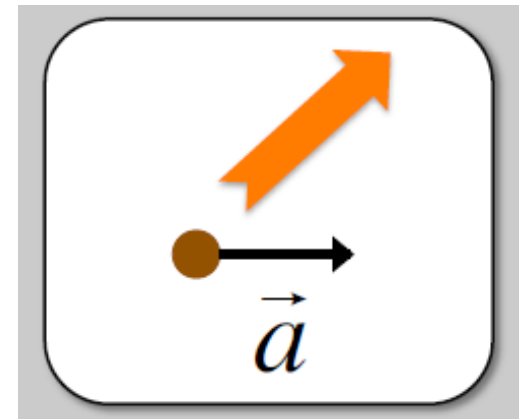


# Mechanisms of production of electromagnetic radiation

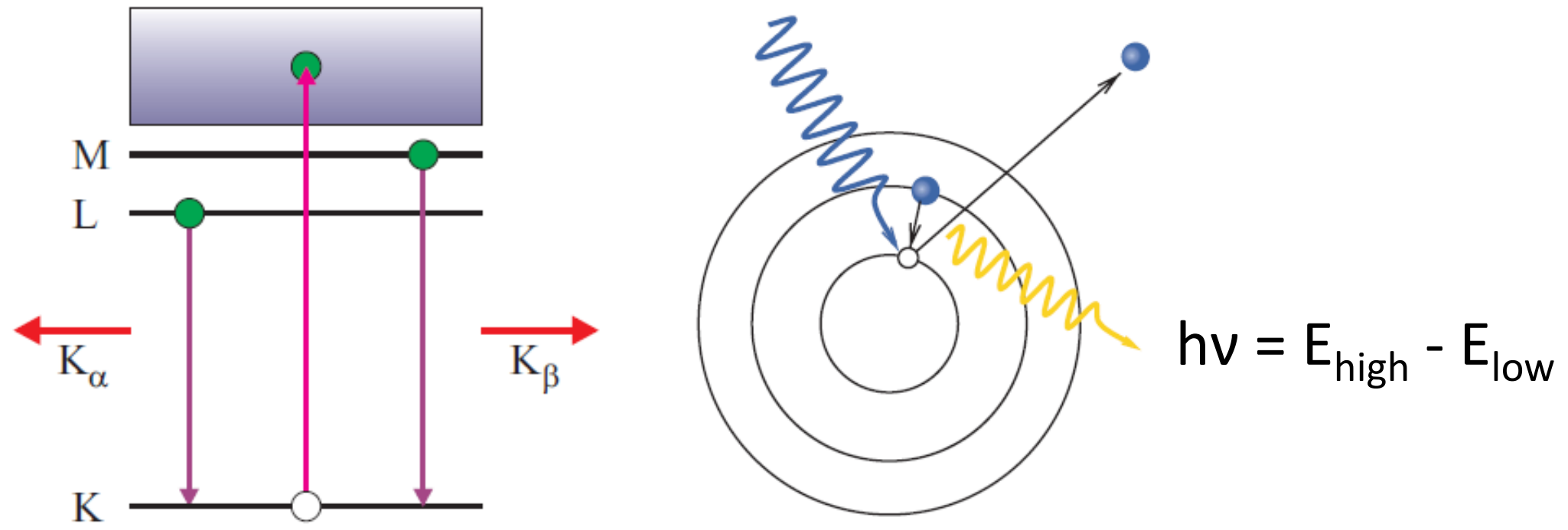
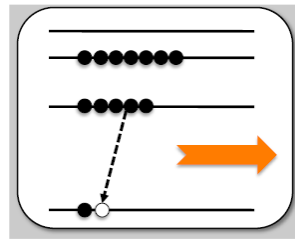
1) Emission as effect of quantum transitions

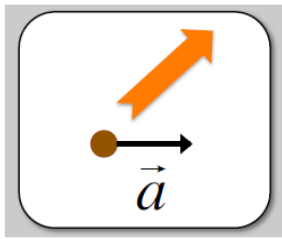


2) Emission from accelerated electric charges



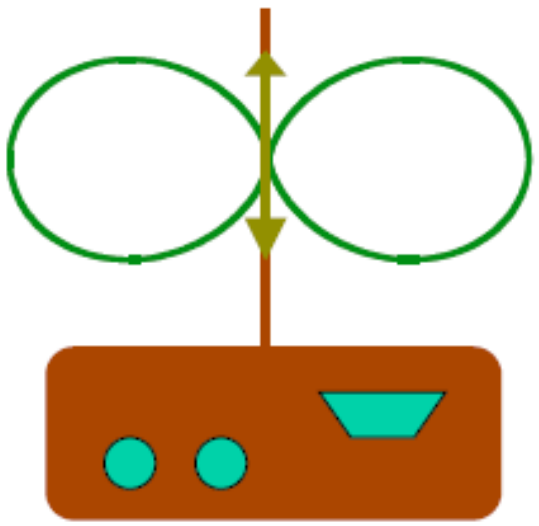
# 1) Emission as effect of quantum transitions





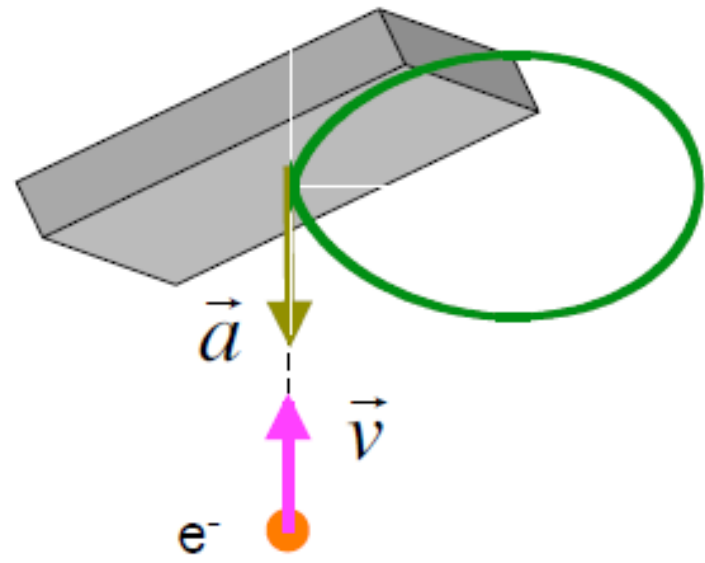
# 2) Emission from accelerated charges

Oscillating motion



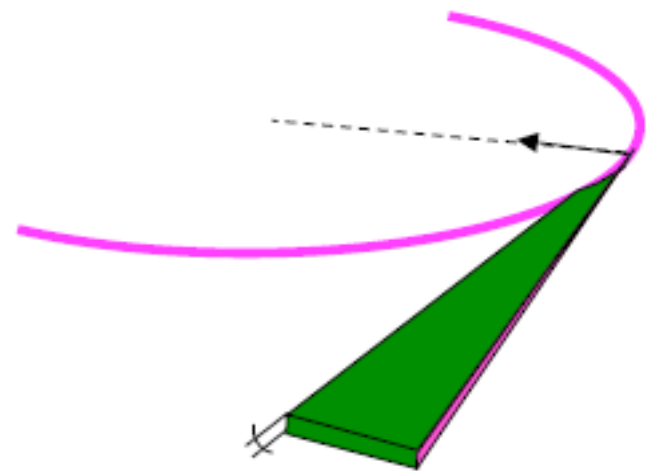
Broadcasting antenna

Average linear deceleration



Bremsstrahlung in x-ray tubes

Relativistic circular motion



Synchrotron radiation



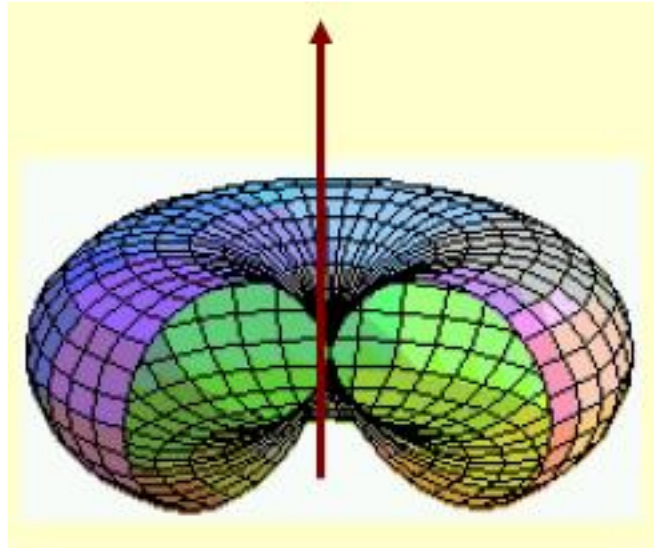
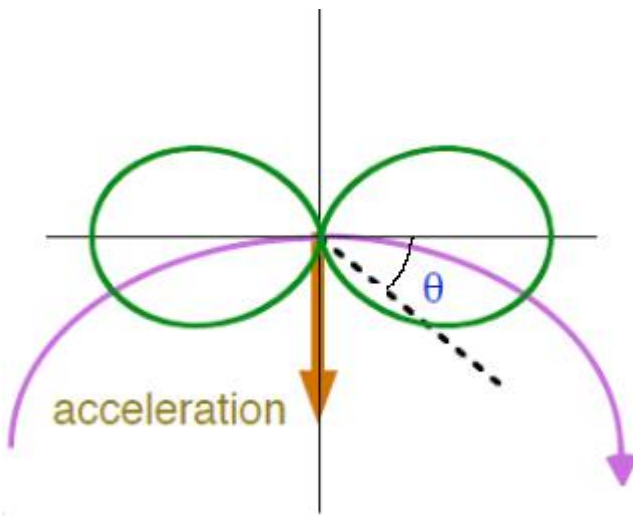
# Emission from accelerated charges: dipole radiation

An accelerated charged particle emits e.m. radiation

The emission of the radiation is symmetric with respect to the acceleration

The emission is zero in the direction of the acceleration

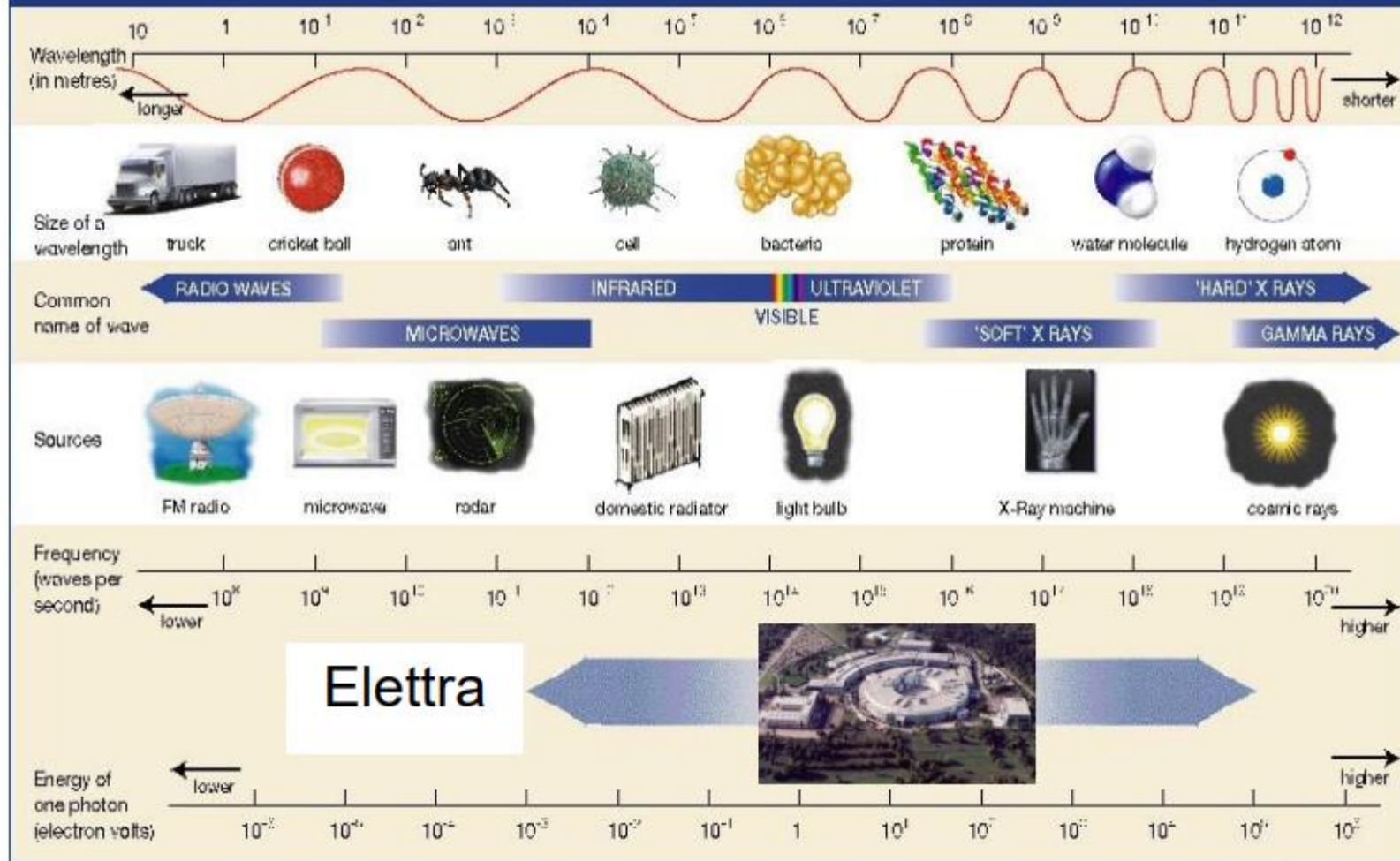
The emission is maximum in the perpendicular plane



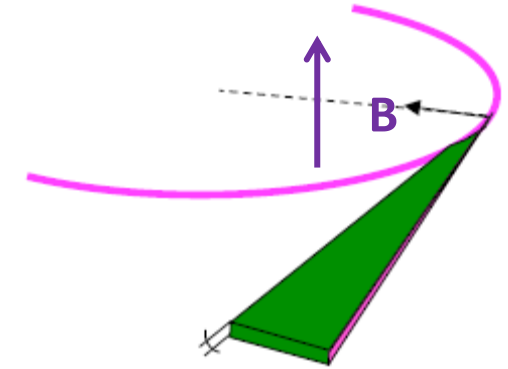
Radiated power

$$P \propto \cos^2 \theta$$

# The Electromagnetic Spectrum



# Synchrotron Radiation



Definition:

Radiation emitted by charged particles moving at **relativistic speed** forced by magnetic fields to follow curved trajectories

The magnetic field, perpendicular to the direction of the electron motion centripetally accelerate the electrons.

*At relativistic speeds the Lorentz factor has to be taken into account*

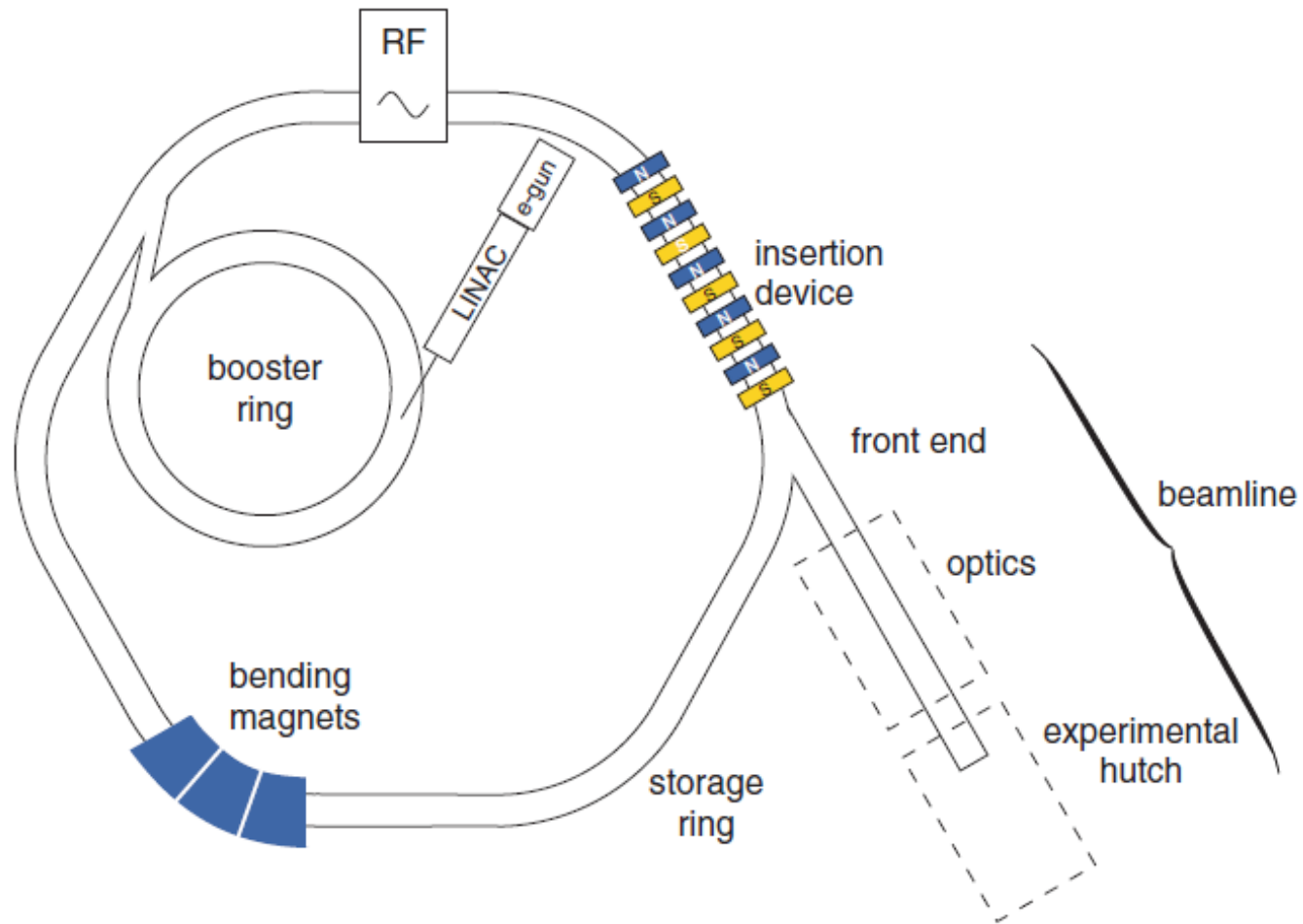
$$\mathcal{E}_e = \frac{m_0 c^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$\mathcal{E}_e = \gamma m_0 c^2$$





# Storage rings



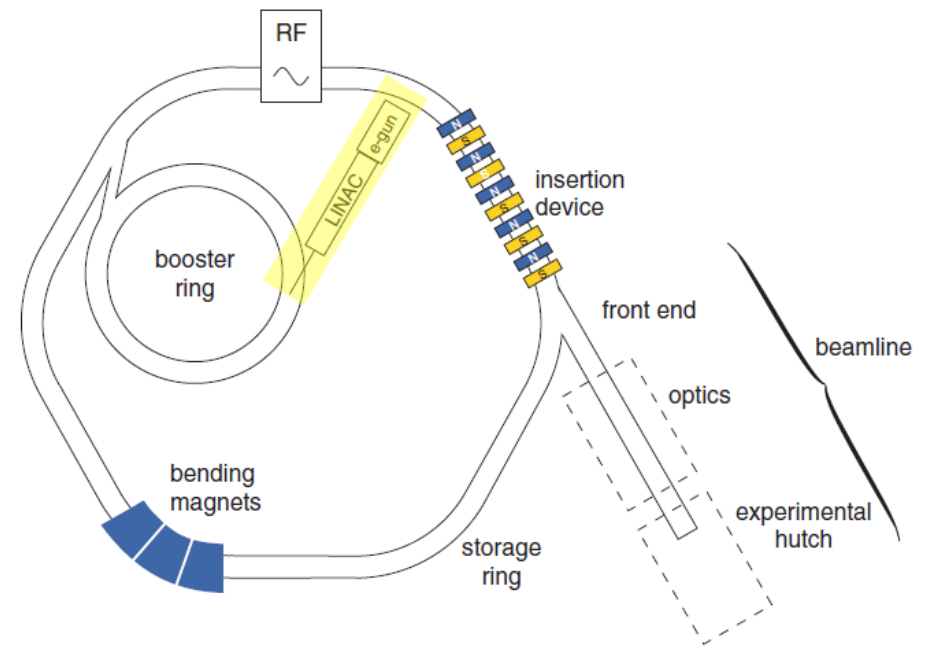
5 main components

# Electron gun and linac

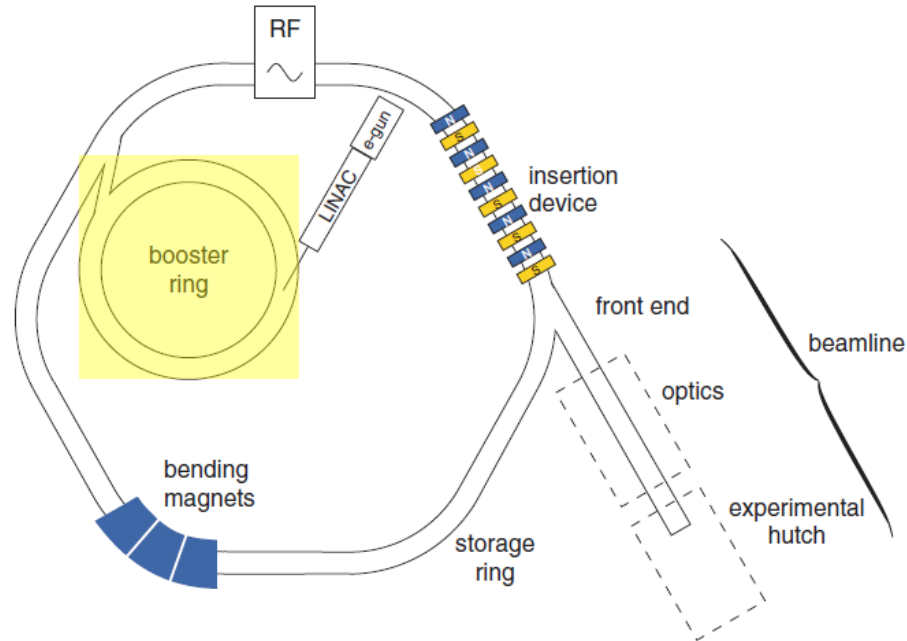
Electrons are generated by thermoionic emission from a hot filament in an electron gun

The electrons are accelerated using a Linac to about 100 MeV

A regular supply of electrons is required, as they are always being lost in the machine, due to collisions with residual gas particles in the storage ring



# Booster ring



Electrons are injected from the linac and further accelerated.

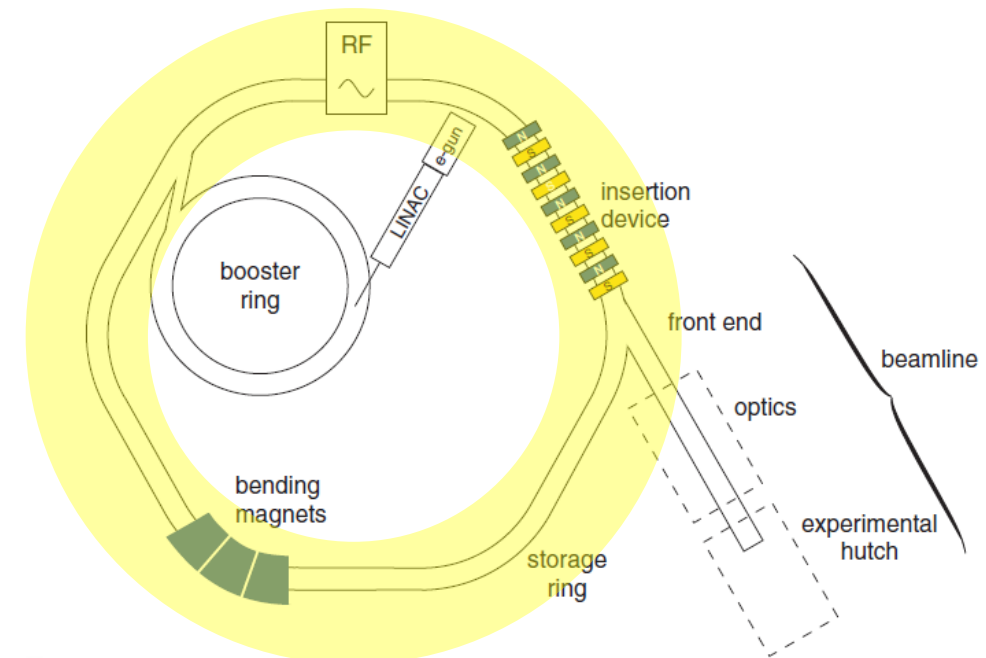
They may either be accelerated to the energy of the electrons in the main storage ring, or (less commonly, especially for modern facilities) to a somewhat lower energy.

# Storage ring

Electrons are injected from the booster periodically so that the specified current is maintained. This is done when the current drops to about  $1 - 1/e \approx 70\%$  or more often in case of top up mode

The storage ring contains the electrons and maintains them on a closed path by the use of an array of magnets, commonly referred to as the 'magnet lattice' of the ring.

The electrons have kinetic energies measured in GeV, and their velocities are highly relativistic, that is, only very marginally less than the velocity of light.



# Storage ring: magnet lattice



**Bending, dipole-magnets**  
They cause the electrons to change their path and thereby follow a close path



**Quadrupole-magnets**  
They are used to focus the electron beam and for Coulomb repulsion between electrons

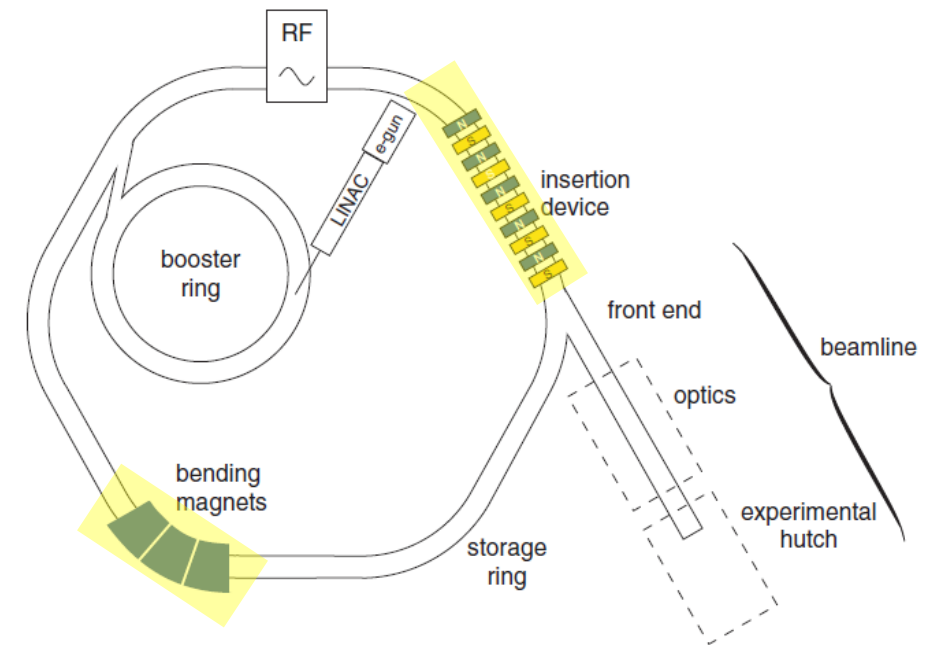


**Sextupole-magnets**  
They correct the chromatic aberration that arise from focusing by the quadrupoles

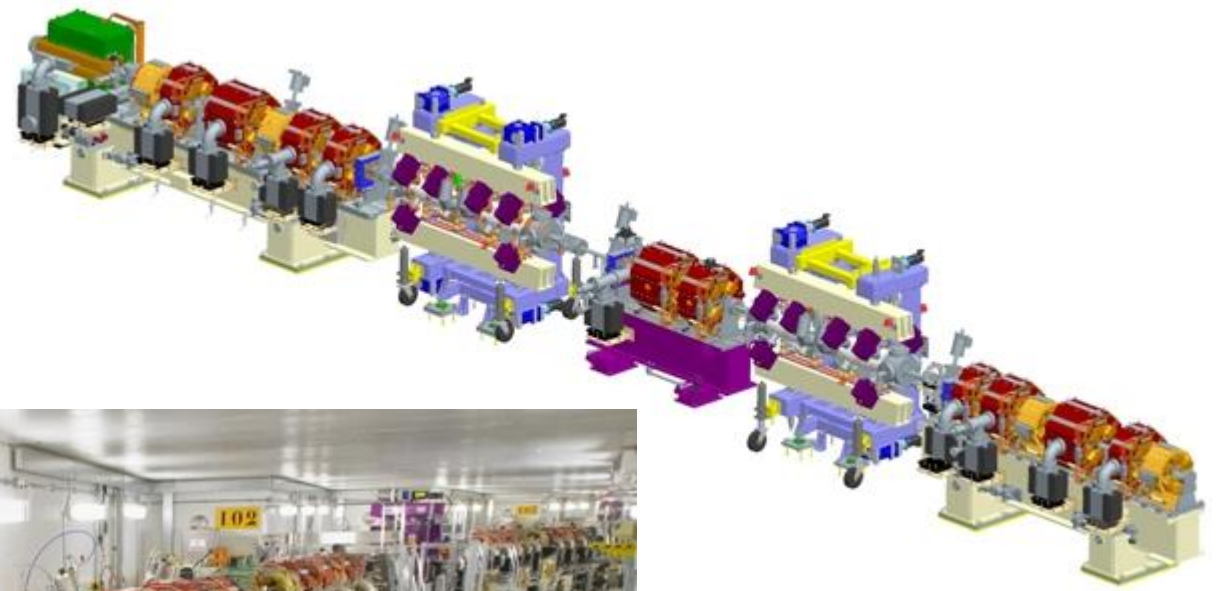
# Storage ring: BMs and IDs

The ring has a structure consisting of arced sections containing bending magnets (BMs) and straight sections used for insertions devices (IDs), which generates the most intense SR.

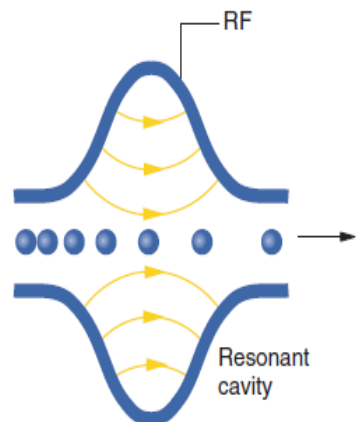
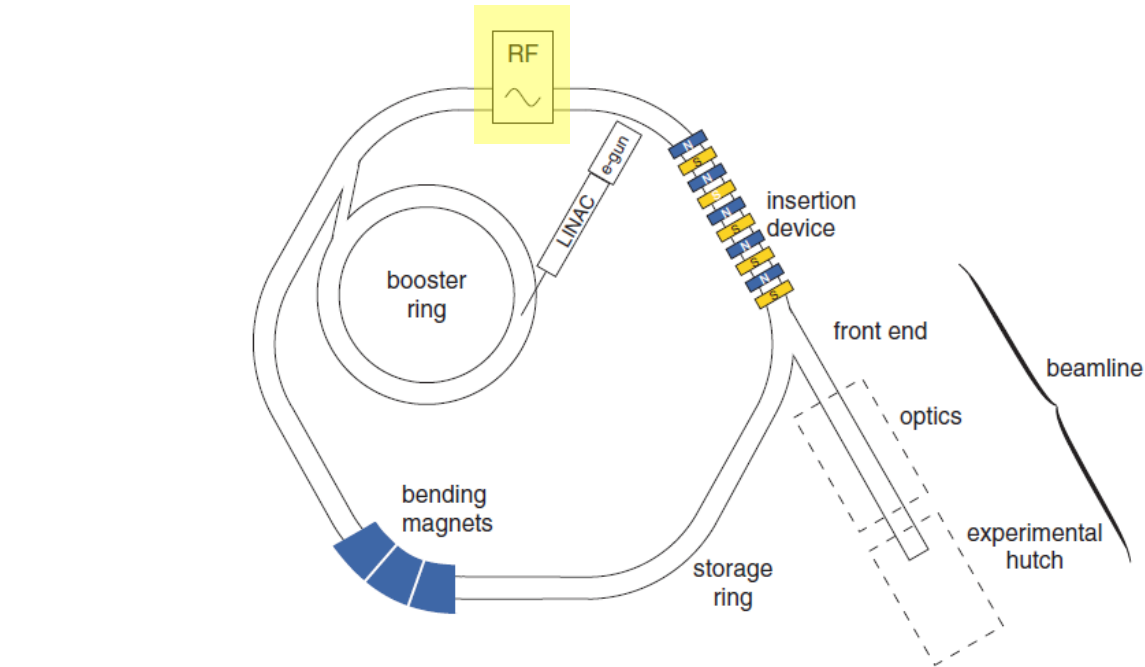
The BMs, used to deflect the electrons round the arced sections that connect the straight sections are also often used to provide BM radiation – although their brilliance is significantly lower than that produced by IDs, even monochromated BM-radiation is still orders of magnitude more intense than that can be provided by laboratory-based sources.







# Radio frequency supply



A periodic electric field is applied in the direction of the electrons

The kinetic energy of the electrons dissipated due to emission of radiation at BMs and IDs must be replenished before they spiral into the inner wall of the storage ring.

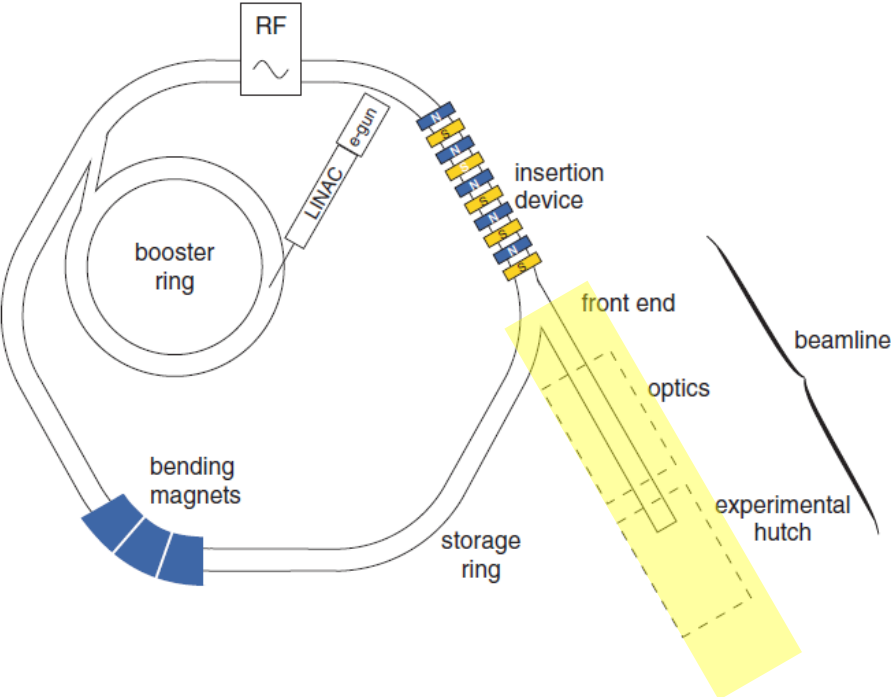
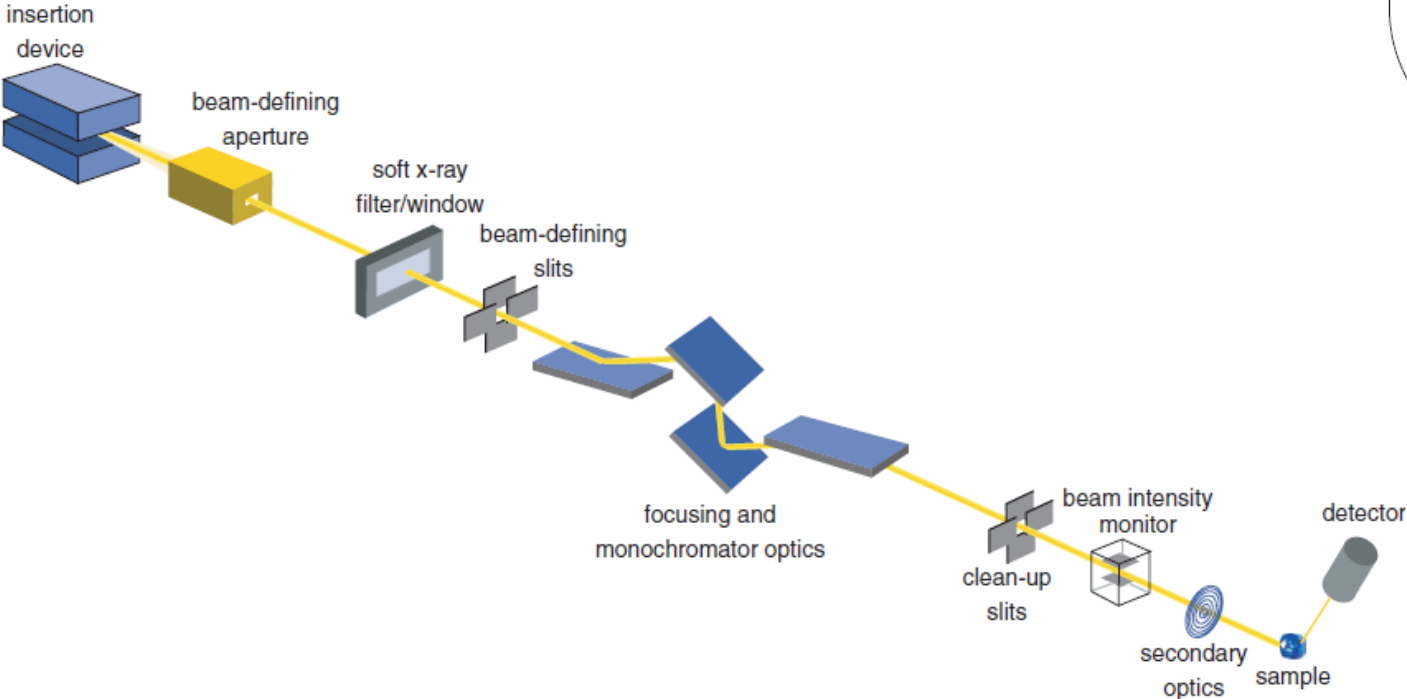
This is achieved by giving them a small boost at every turn as they pass through a radio-frequency cavity (klystron).





# Beamlines

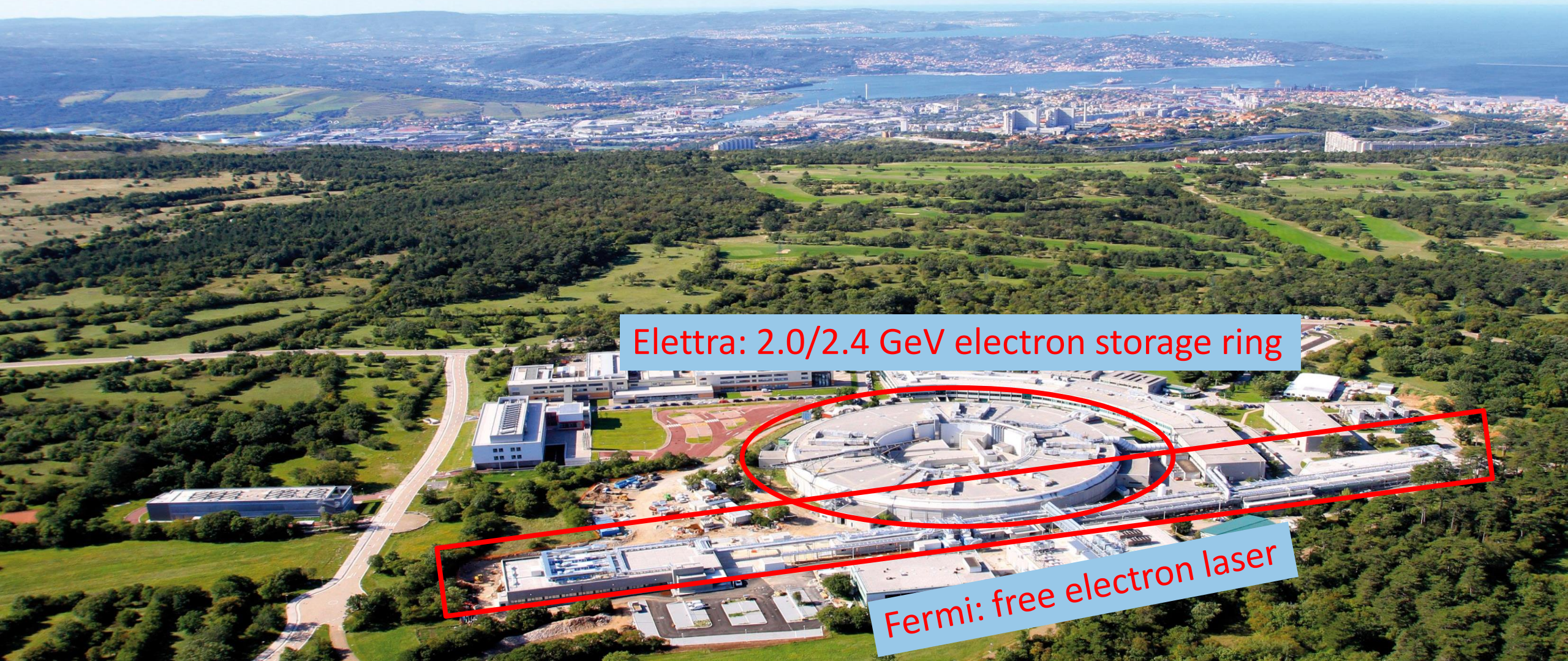
They run off tangentially to the storage ring, along the axes of the IDs and tangentially to DMs







Elettra  
Sincrotrone  
Trieste



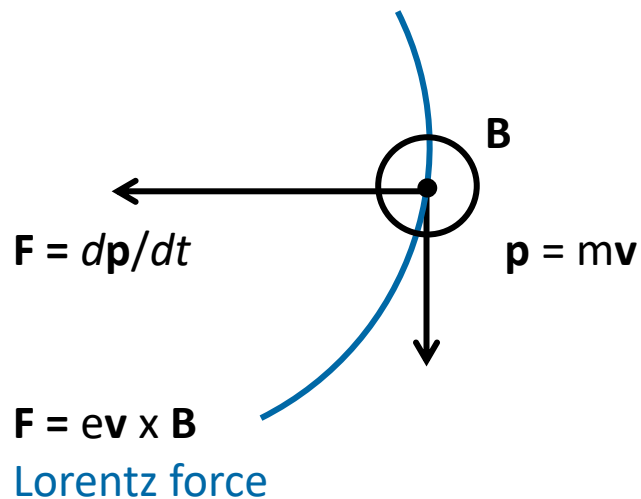
Elettra: 2.0/2.4 GeV electron storage ring

Fermi: free electron laser



# Equation of motion - 1

An electron moving in a magnetic field radiates energy



An electron of momentum  $\mathbf{p} = m\mathbf{v}$  moving in a constant magnetic field  $\mathbf{B}$  experiences the Lorentz force  $\mathbf{F} = d\mathbf{p}/dt = e\mathbf{v} \times \mathbf{B}$ . In response to this force the electron accelerates and moves in a circular orbit in a plane perpendicular to  $\mathbf{B}$ .

The Lorentz force, being perpendicular to the motion, does no work and *cannot change the energy of the electrons*, but it does cause a centripetal acceleration that changes the direction of the velocity

# Equation of motion - 2

Considering the relativistic formulae:

$$\mathbf{p} = \gamma m_0 \mathbf{v}$$
$$\mathbf{F} = \frac{d}{dt} (\gamma m_0 \mathbf{v}) = e \mathbf{v} \times \mathbf{B}$$

They are perpendicular

$$\gamma m_0 \frac{d}{dt} \mathbf{v} = evB$$

Centripetal acceleration  $\swarrow$

$$\gamma m_0 \frac{v^2}{\rho} = evB$$

Orbital radius  $\longleftarrow$

Since  $v \approx c$ ,

$$\gamma m_0 \frac{c^2}{\rho} = ecB$$



# Equation of motion - 3

$$\rho = \frac{\gamma m_0 c}{eB}$$

Since  $\gamma = \frac{\varepsilon}{m_0 c^2}$

$$\rho = \frac{\varepsilon}{ceB}$$

In practical units:

$$\rho[\text{m}] = 3.3 \frac{\varepsilon[\text{GeV}]}{B[\text{T}]}$$

As typical magnetic field strengths of bending magnets are 1 Tesla and storage rings electron energies are normally of the order of few GeV, the bending radius is typically a few meters

High energy synchrotron are places in big storage rings



ESRF : 6 GeV (845 m)



APS : 7 GeV



SPring8 : 8 GeV (1435 m)













# SR facilities: map



- 47 SR research facilities
- 23 countries
- more than 30000 scientists and engineers  
(including thousand of students)

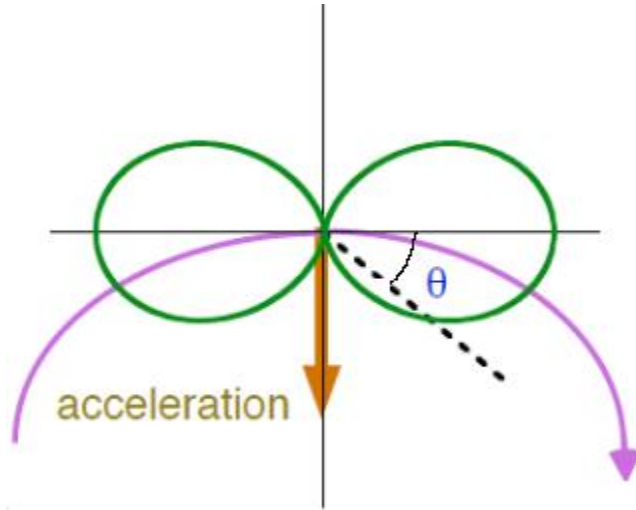
[www.lightsource.org](http://www.lightsource.org)

# Characteristics of the SR

- 1) Collimation
- 2) Broad band
- 3) Polarization
- 4) Time structure
- 5) Coherence

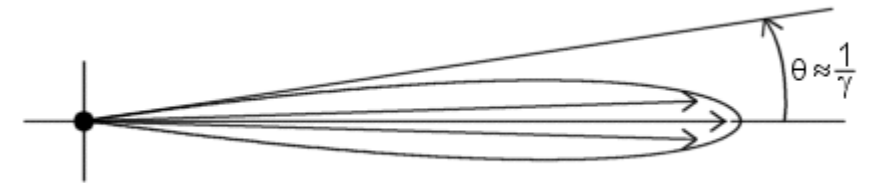


# 1) Collimation



In the **moving frame of the electron** ( $v \ll c$ ) the power emitted by an accelerated particle has a characteristic two-lobe distribution around the direction of the acceleration.

In the **laboratory frame of reference** ( $v \sim c$ ) all the emitted power is beamed into a narrow cone in the direction of motion.



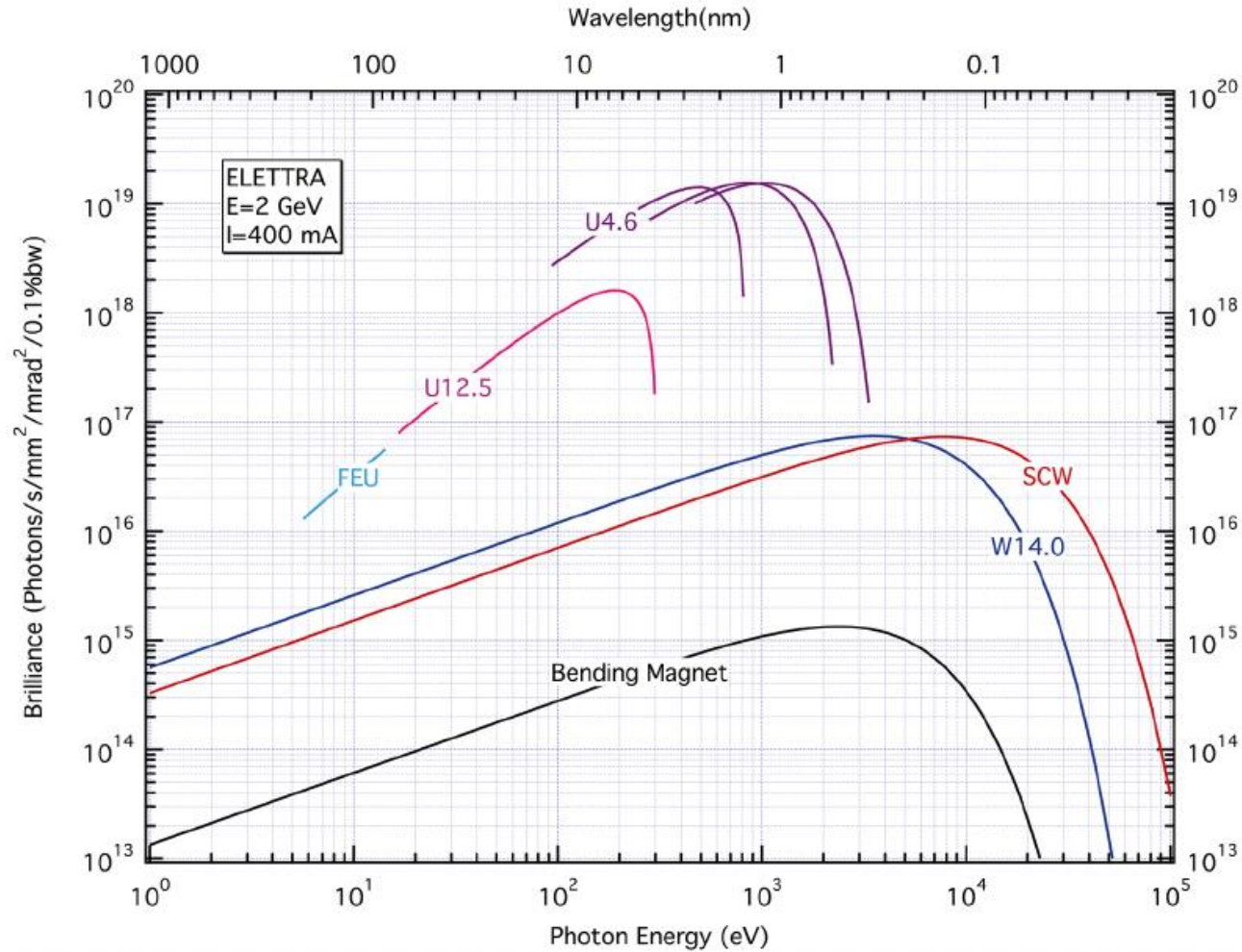
All the forward power is radiated in a beam of angle  $\frac{2}{\gamma}$

# More on collimation of SR

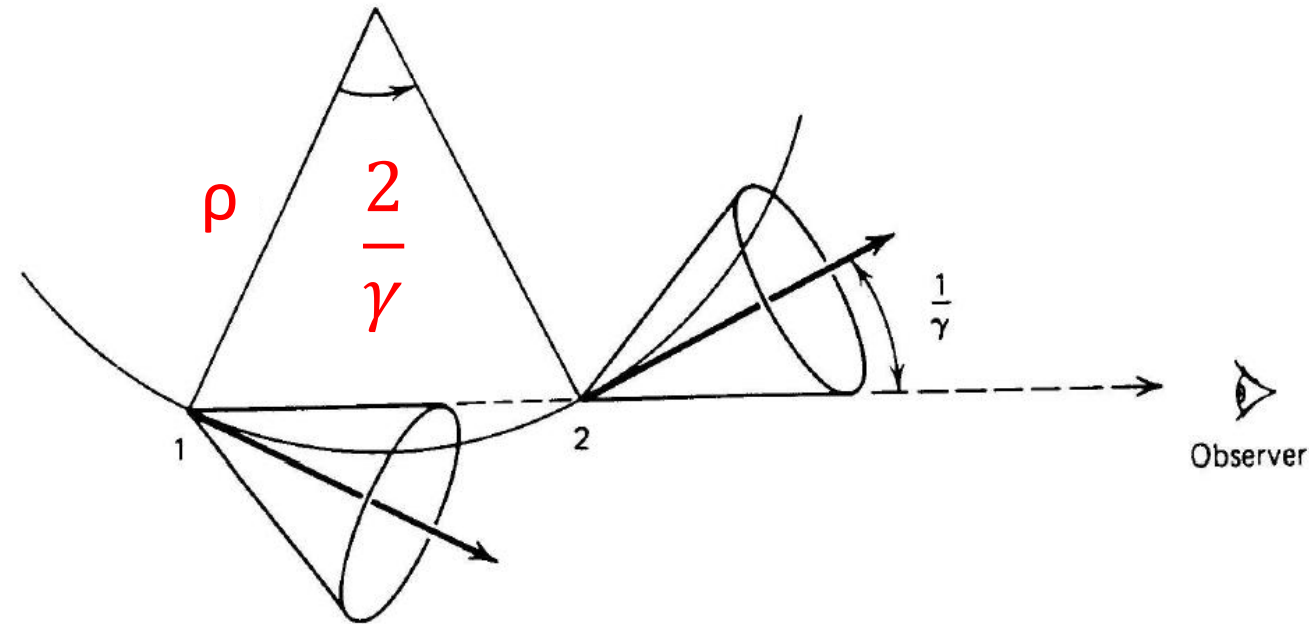


- The collimation of synchrotron radiation is a **direct consequence of the relativistic speed of the electrons**
- The collimation conserves energy: the emission found in the electron frame is now concentrated in a small cone.
- This affects a fundamental figure of merit for light sources: the “**brightness**”. (The brightness is proportional to the emitted flux divided by the angular spread and by the source size.)
- Very high fluxes on very small area also at distances of tens of meters from the storage rings.

## 2) Broadband emission



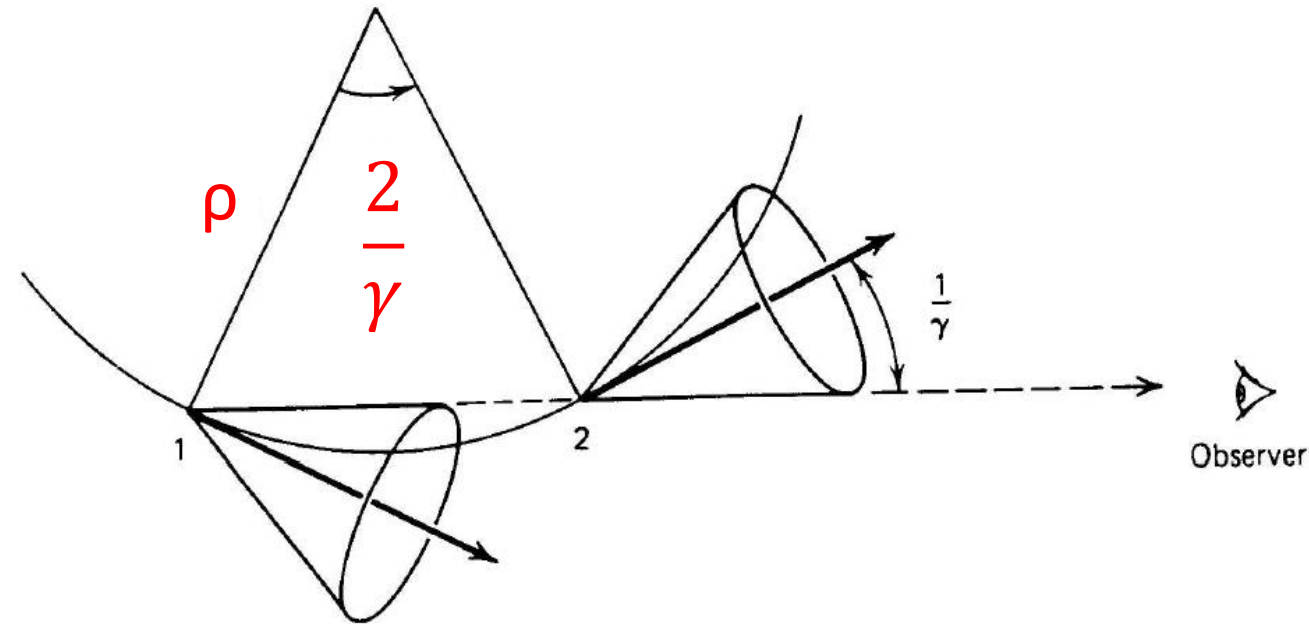
# Broadband emission: pulse duration



The time of emission of radiation is given by the time the electron runs at speed  $v$  along the arc from 1 to 2

However the radiation emitted at 1 will reach the observer with a delay given by the fact that the radiation must travel (at speed  $c$ ) to the observer

# Broadband emission: pulse duration



The pulse duration  $\Delta\tau$ :

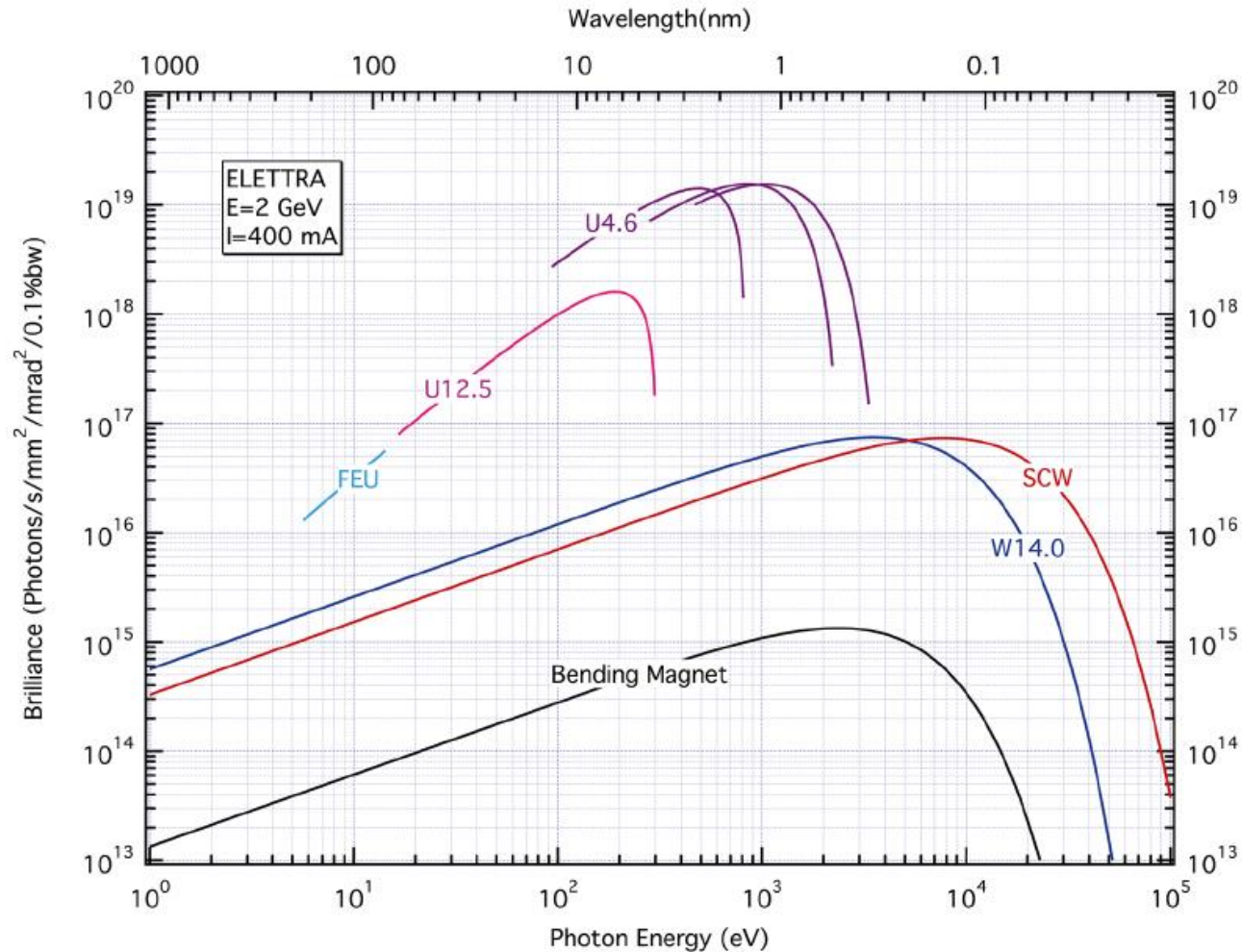
$$\Delta\tau = \frac{\text{arc length}}{v} - \frac{\text{radiation path}}{c}$$

$$\Delta\tau = \frac{\rho 2/\gamma}{v} - \frac{2\rho \sin(1/\gamma)}{c}$$

$$\Delta\tau \approx \frac{\rho}{c\gamma^3}$$



## 2) Broadband emission



The Fourier theorem relates this pulse duration  $\Delta\tau$  to  $\Delta\nu$

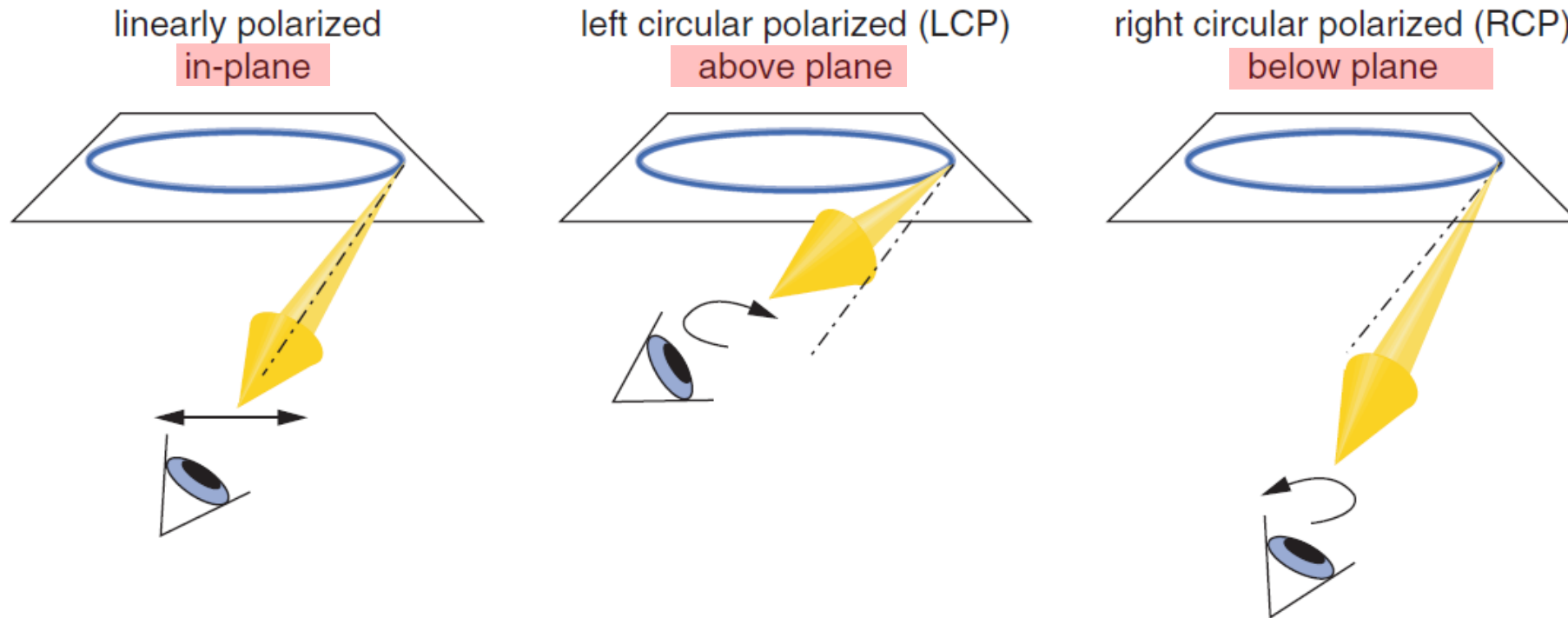
$$\Delta\nu \approx \frac{1}{2\pi\Delta\tau}$$

$$\Delta E \approx \frac{hc\gamma^3}{2\pi\rho}$$



# 3) Polarization

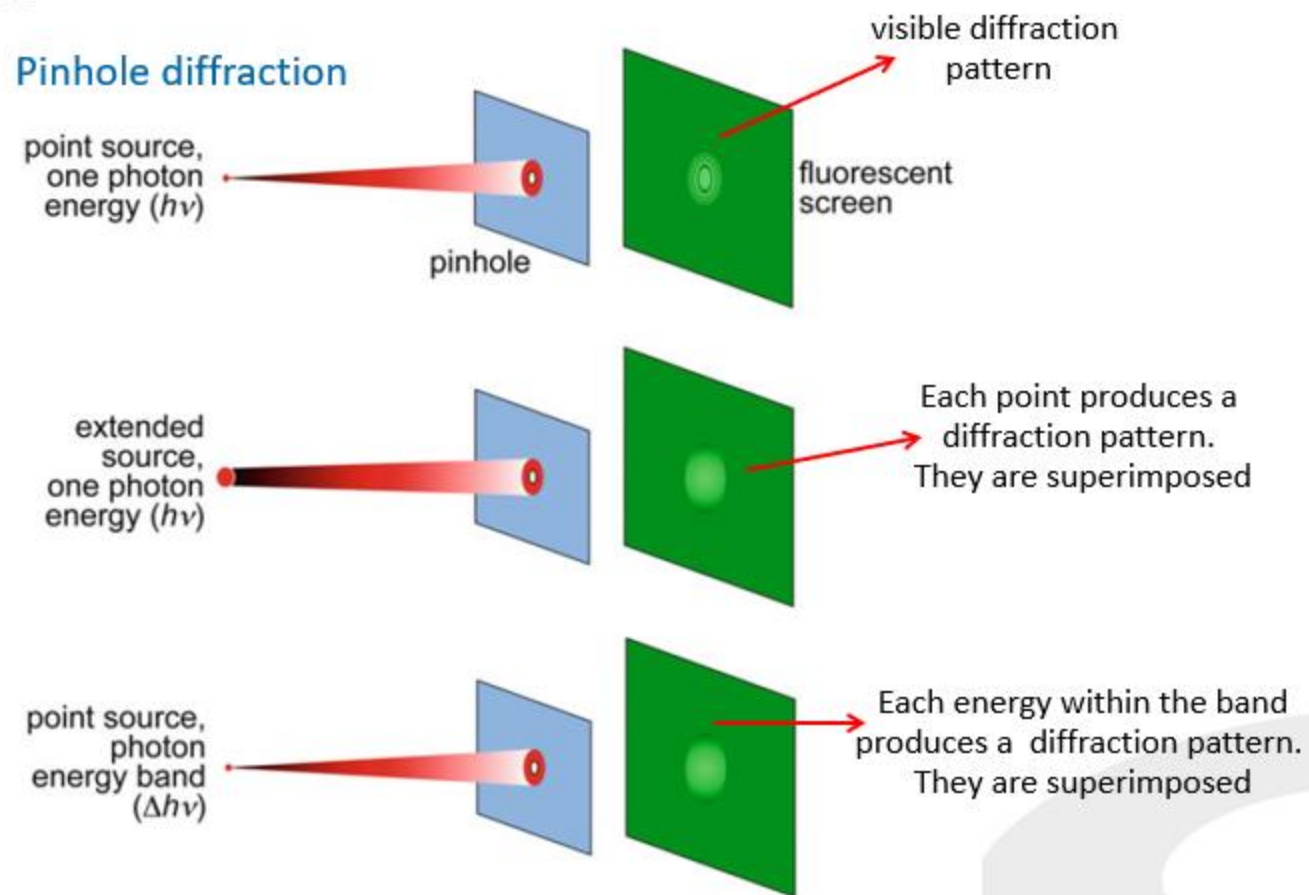
The polarization of the x-rays emerging from a storage ring depends on the line of sight



The electrons appear to oscillate in the horizontal plane

The electrons appear to execute an elliptical orbit in clockwise/anticlockwise direction.

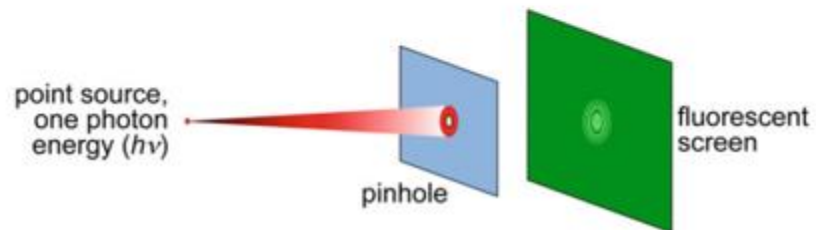
Coherence enables to observe phenomena such as diffraction and interference



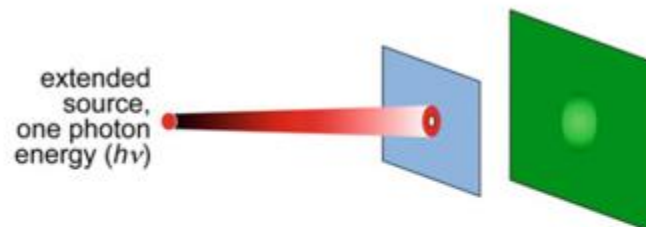
Longitudinal or time coherence: linked to the spectral bandwidth

Lateral or space coherence: linked to the source geometry

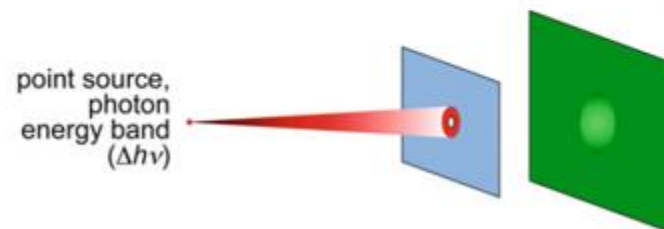
time and space coherent



time coherent



space coherent



Criterion for minimal longitudinal (temporal) coherence:

$$\frac{\Delta E}{E} < 1$$

For broadband emission (bending magnets):

- light can be monochromatized ( $\Delta E/E=10^{-4}$  for Si(111))

For non broadband emission (undulators):

- sources naturally longitudinally coherent

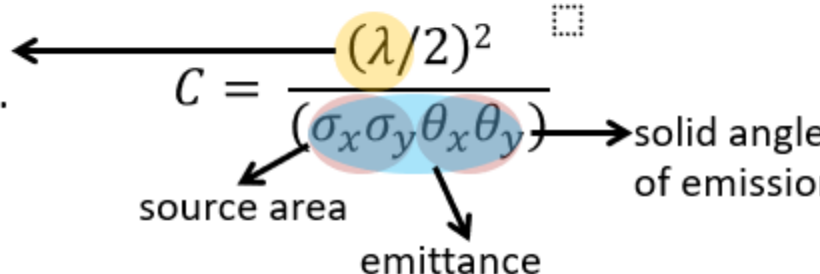


- The **fraction of lateral coherent power** characterizes the level of lateral coherence of the source

C increases with the square of the  $\lambda$ .

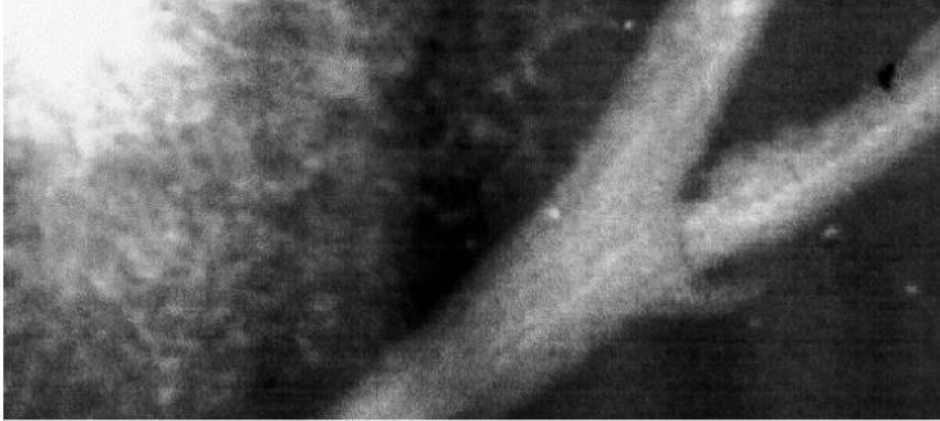
$$C = \frac{(\lambda/2)^2}{(\sigma_x \sigma_y \theta_x \theta_y)}$$

source area      emittance      solid angle of emission

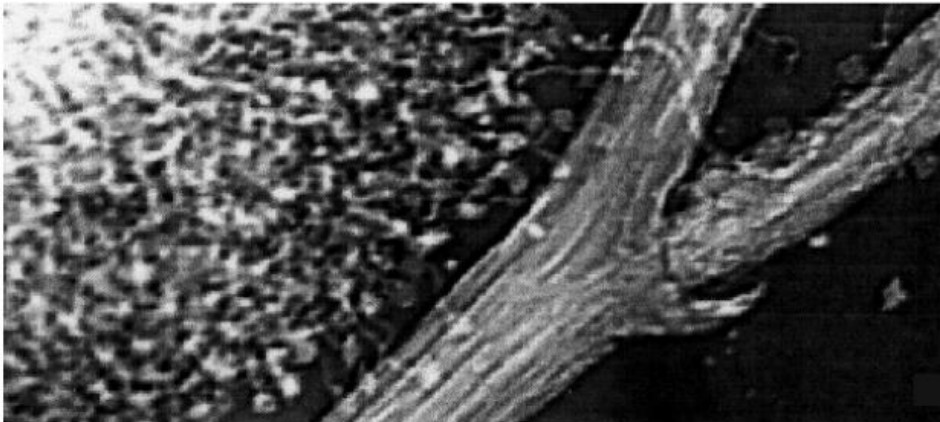


- All types of coherence are more difficult to achieve at high photon energies (x-rays) than at lower photon energies like visible or infrared
- The improved geometry of the source lead not only to an increase of brightness/brilliance, but also a boost of lateral coherence

# 5) Coherence



Conventional absorption  
image



Coherence- enhanced  
image

# Synchrotron Radiation sources

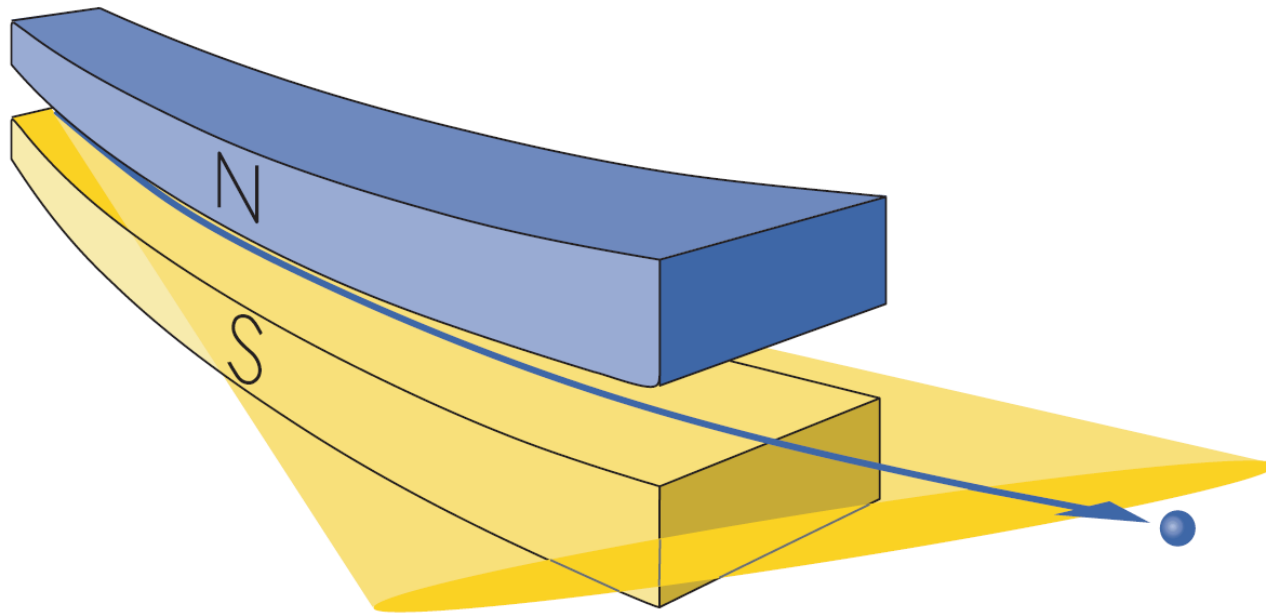


Elettra Sincrotrone Trieste



# Bending magnet: angular distribution

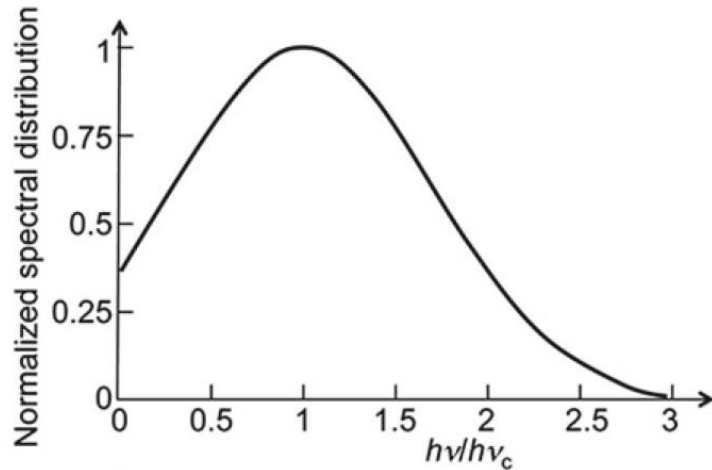
- The primary purpose of the bending magnet is to circulate the electron beam in the storage ring in a close path
- The bending magnet is also used as source for synchrotron radiation



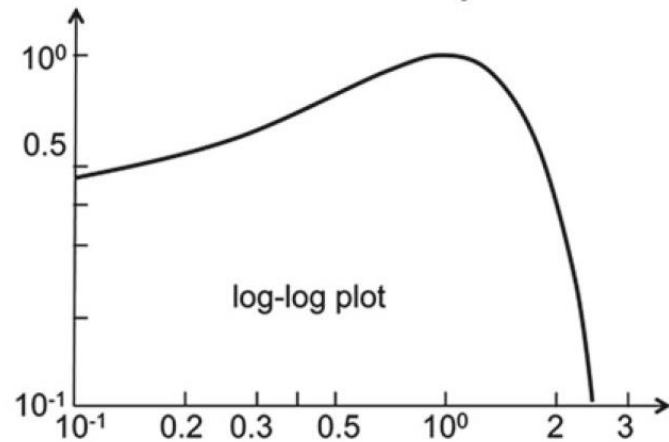
The angular spread of the BM radiation is a flattened cone ( $2/\gamma$  vertically) with an horizontal angle equal to the angular change of the path of the electrons



# Bending magnet: spectral distribution



Linear-linear plot of the approximate lineshape for broadband bending magnet emission



Log-log plot of the approximate lineshape for broadband bending magnet emission

# Bending magnet: critical energy

$$h\nu_{CR}$$

The critical energy is defined by saying that equal amounts of synchrotron radiation energy are emitted at photon energies lower and higher than  $h\nu_{CR}$

In practical units:

$$h\nu_{CR}[\text{keV}] = 0.665\varepsilon^2[\text{GeV}]B[\text{T}]$$

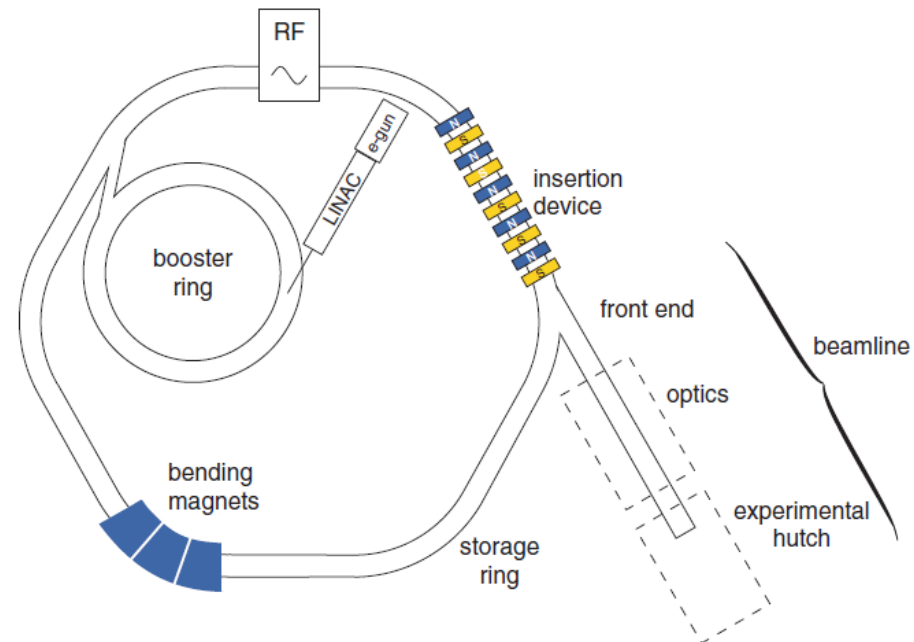
$$h\nu_{CR}[\text{keV}] = 2.21\varepsilon^3[\text{GeV}]/\rho[\text{m}]$$



# Insertion devices - 1

Third generation synchrotrons are characterized by the use of [insertion devices](#) (IDs).

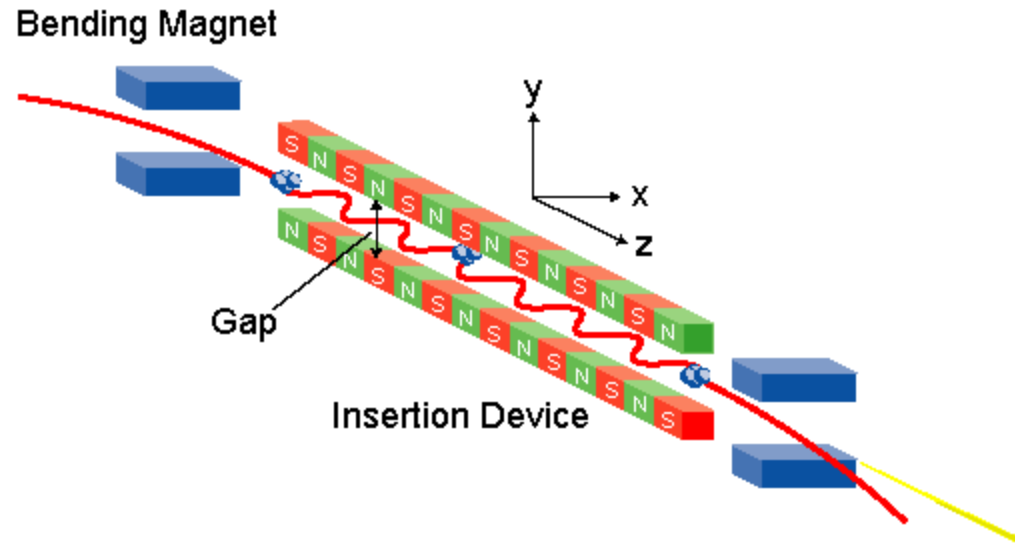
These are placed in the straight sections between the bending magnet arc segments

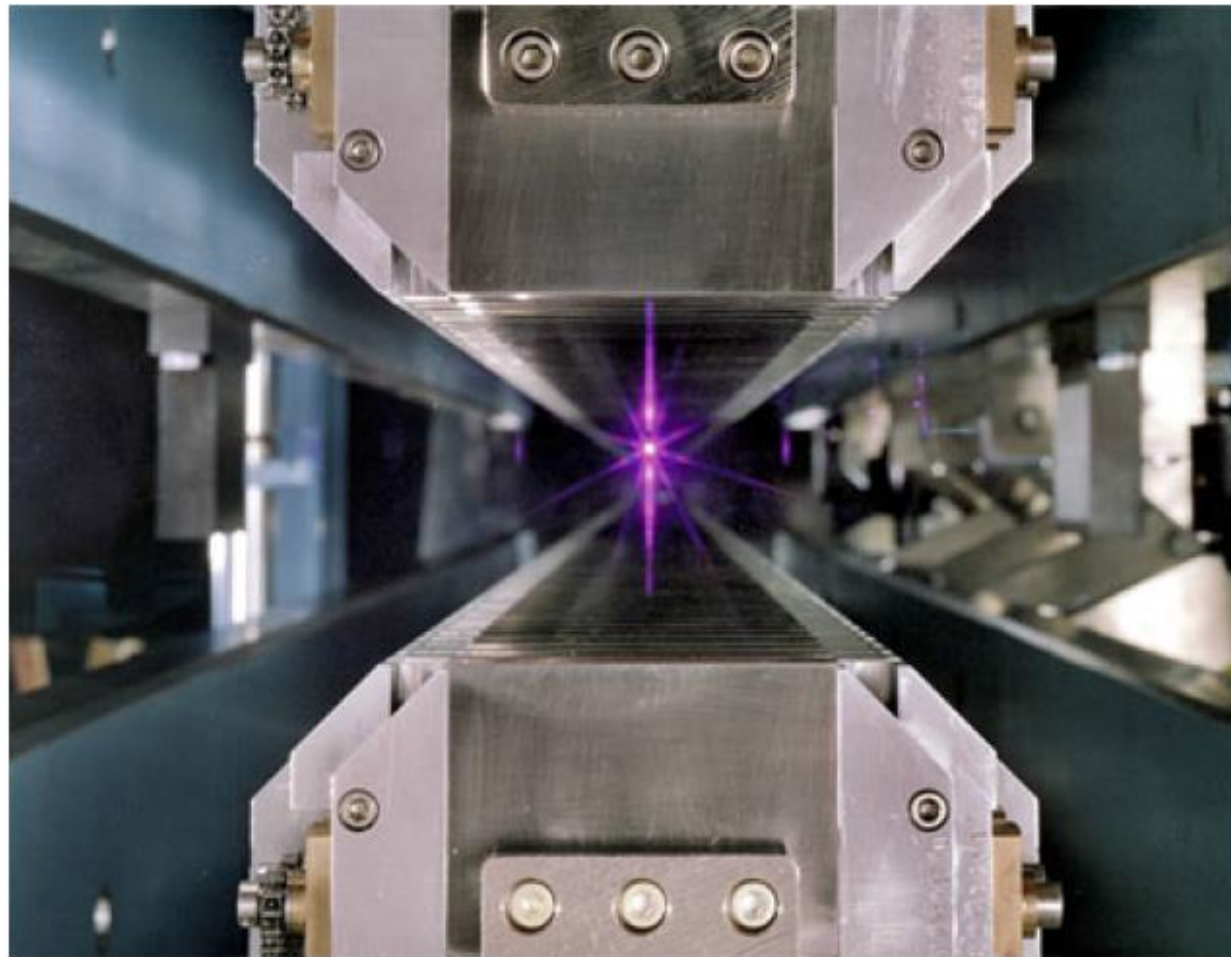
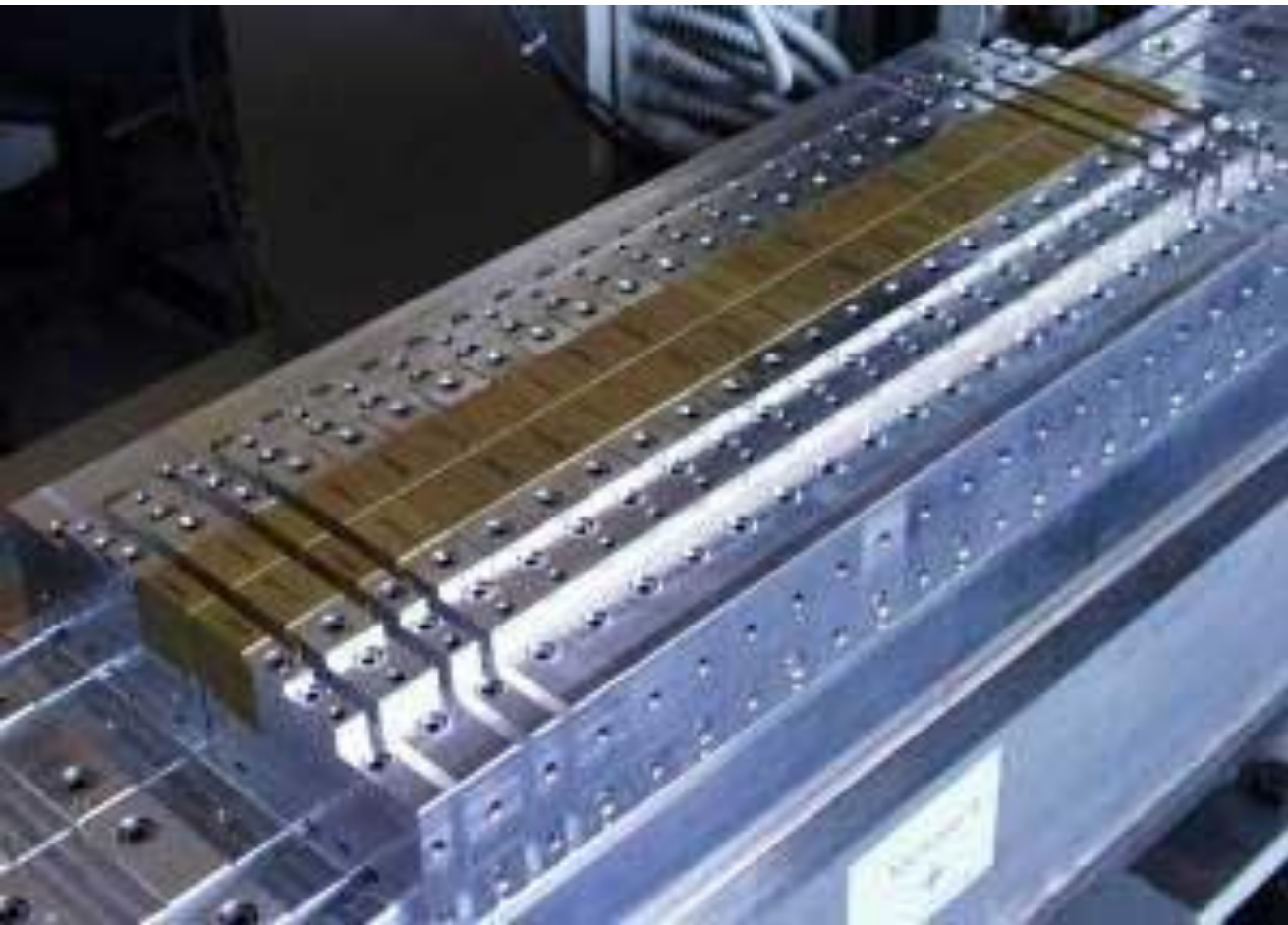


# Insertion devices - 2

Insertion devices are periodic magnetic structures (e.g. permanent magnets: NdFeB).

Passing through such alternating magnetic field structures, **electrons oscillate perpendicularly to the direction of their motion** and therefore emit SR during each individual wiggle





# Insertion devices - 3

Effect of insertion devices:

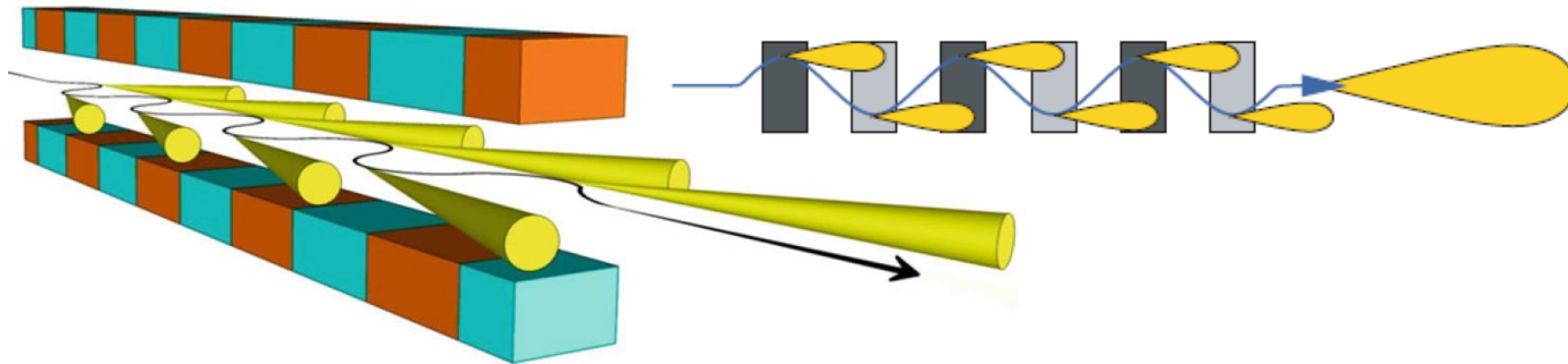
- To shift the critical energy  $h\nu_{CR}$  to higher values due to the smaller bending radius  $\rho$  with respect to the bending magnets
- To increase the intensity of the radiation by a factor related to the number of wiggles induced by many poles of the magnetic structure
- To increase the spectral brightness



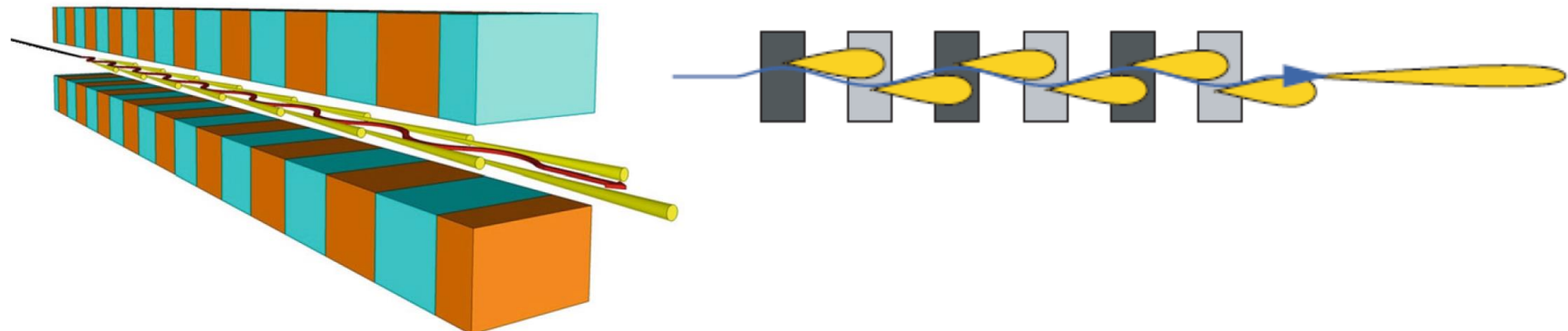
# Wigglers and undulators - 1

Wigglers and undulators are different from one another by the degree to which the electrons are forced to deviate from a straight path

Wigglers



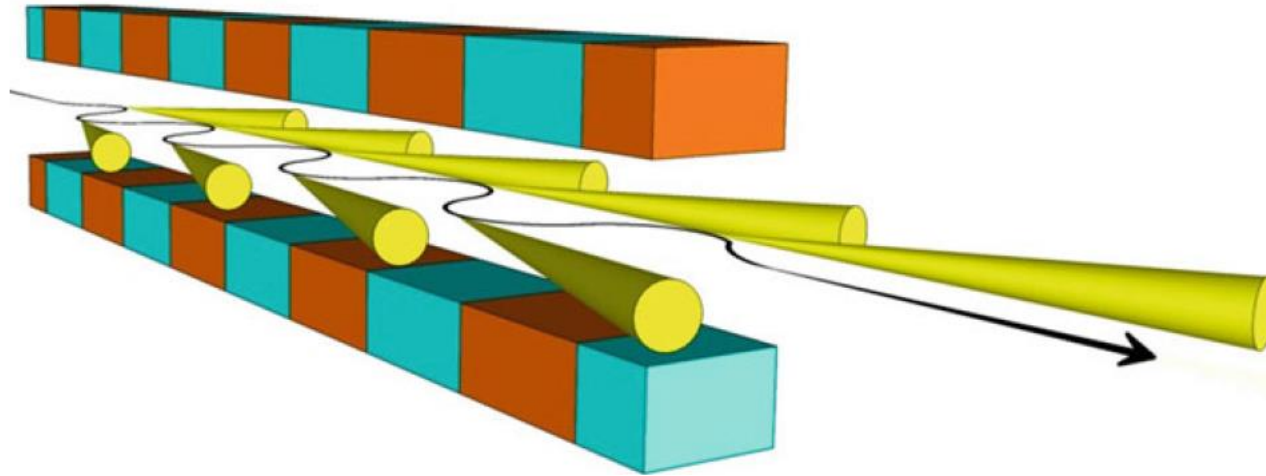
Undulators





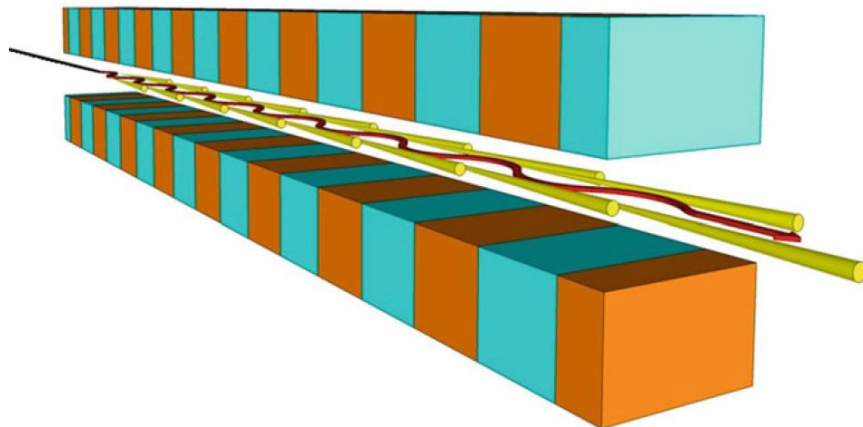
# Wigglers and undulators - 2

## Wigglers



- Just like a BM except:
- Larger  $B \rightarrow$  higher  $h\nu_{CR}$
- More bends  $\rightarrow$  more power

## Undulators



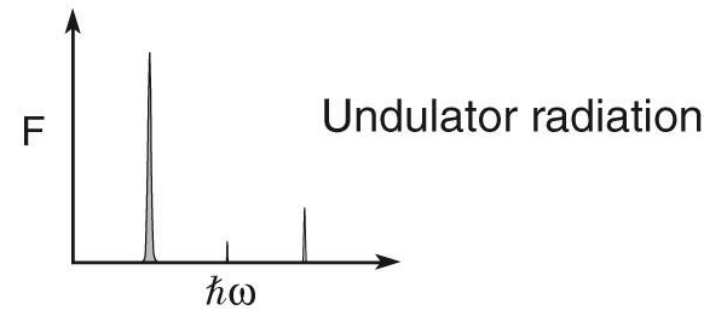
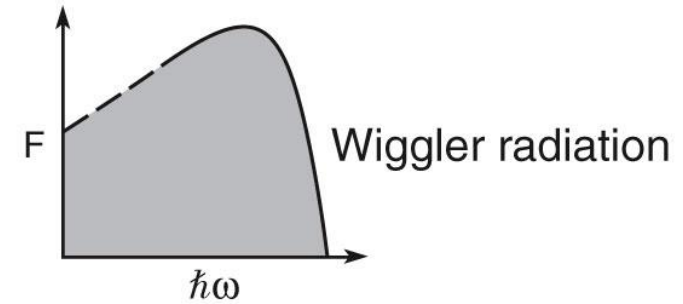
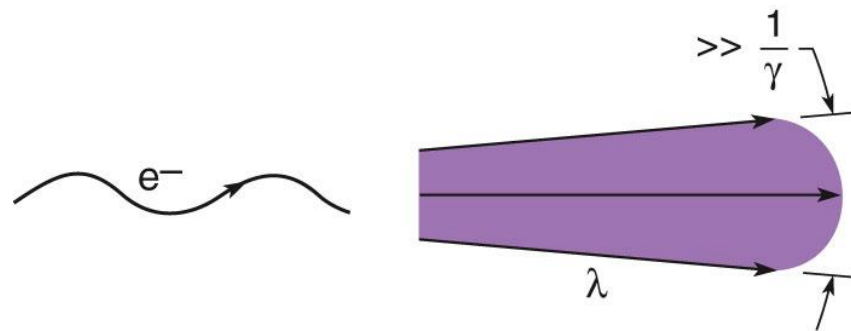
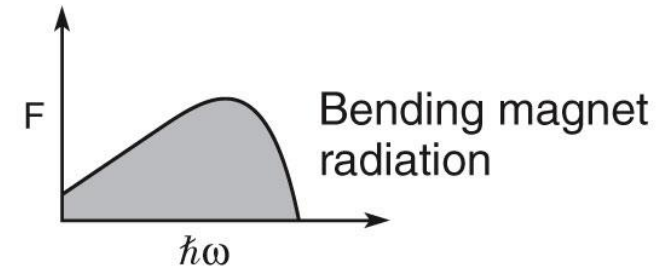
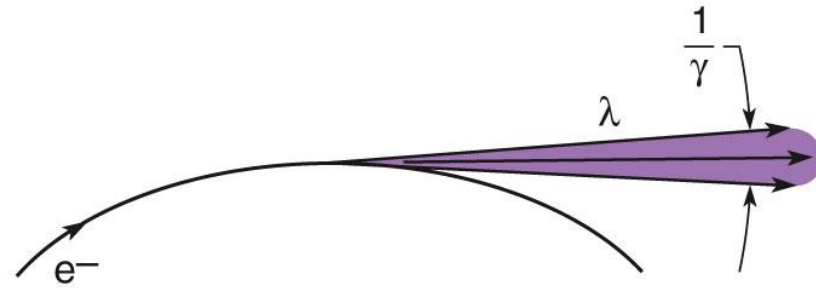
Different from BM:

- Shallow bends  $\rightarrow$  small source
- Interference effect  $\rightarrow$  highly peaked spectrum





# Summary of the 3 SR sources



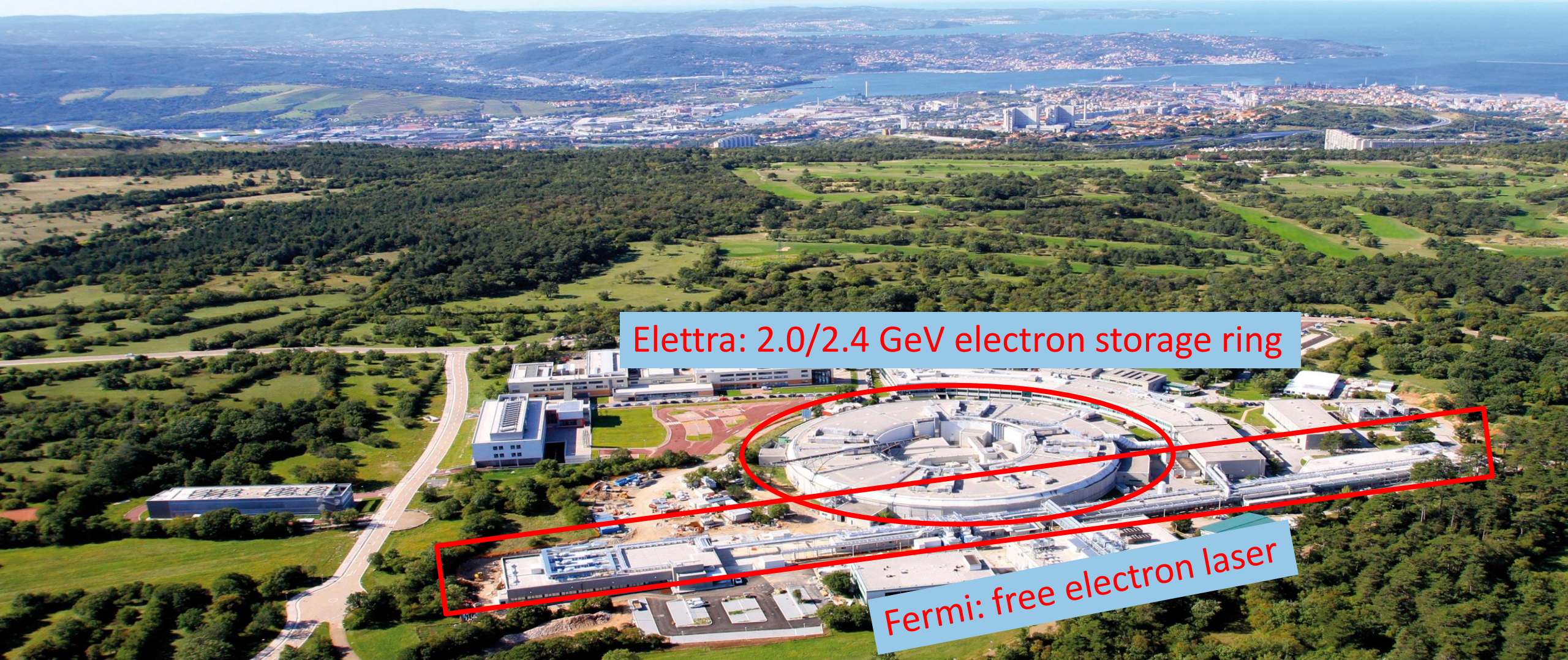
$$\lambda = \frac{\lambda_u}{2n\gamma^2} \left( 1 + \frac{K^2}{2} \right)$$







Elettra  
Sincrotrone  
Trieste

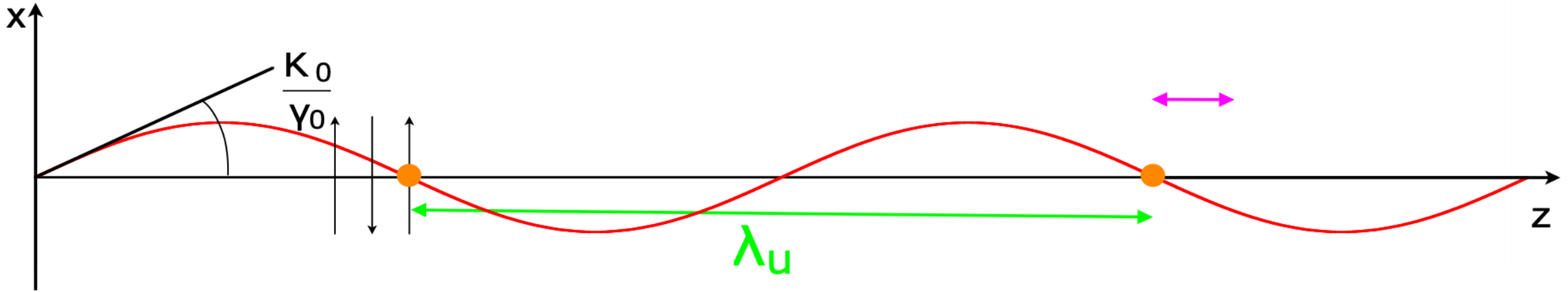


Elettra: 2.0/2.4 GeV electron storage ring

Fermi: free electron laser



# Free electron laser

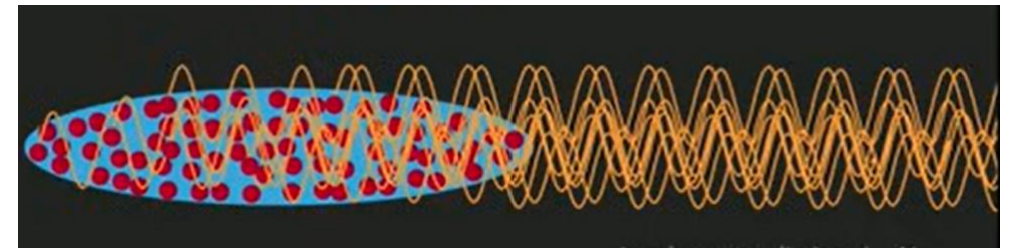
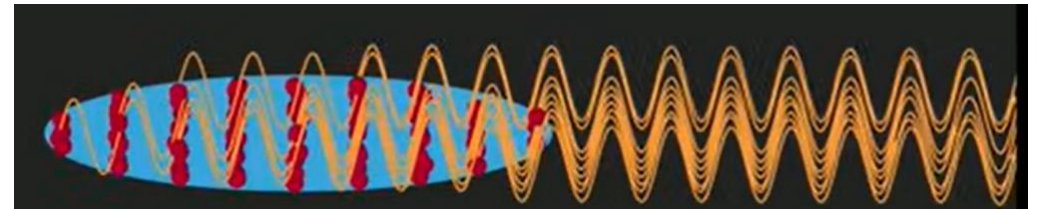
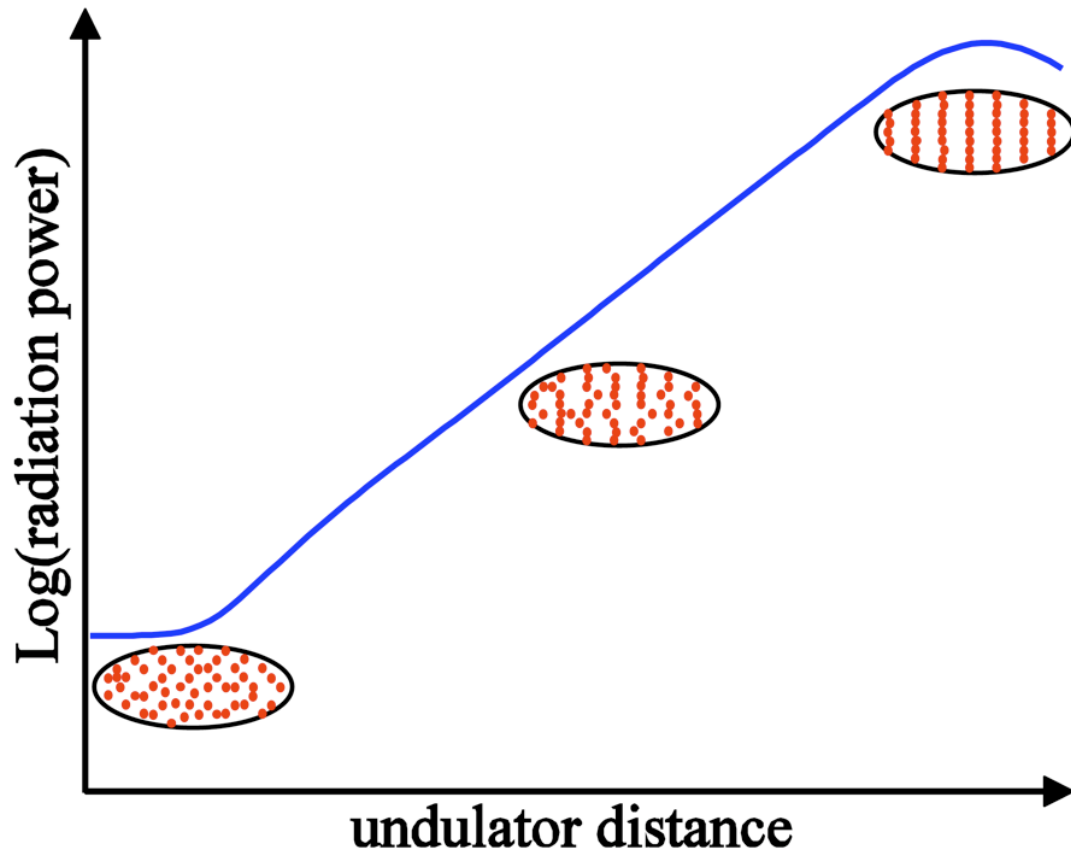


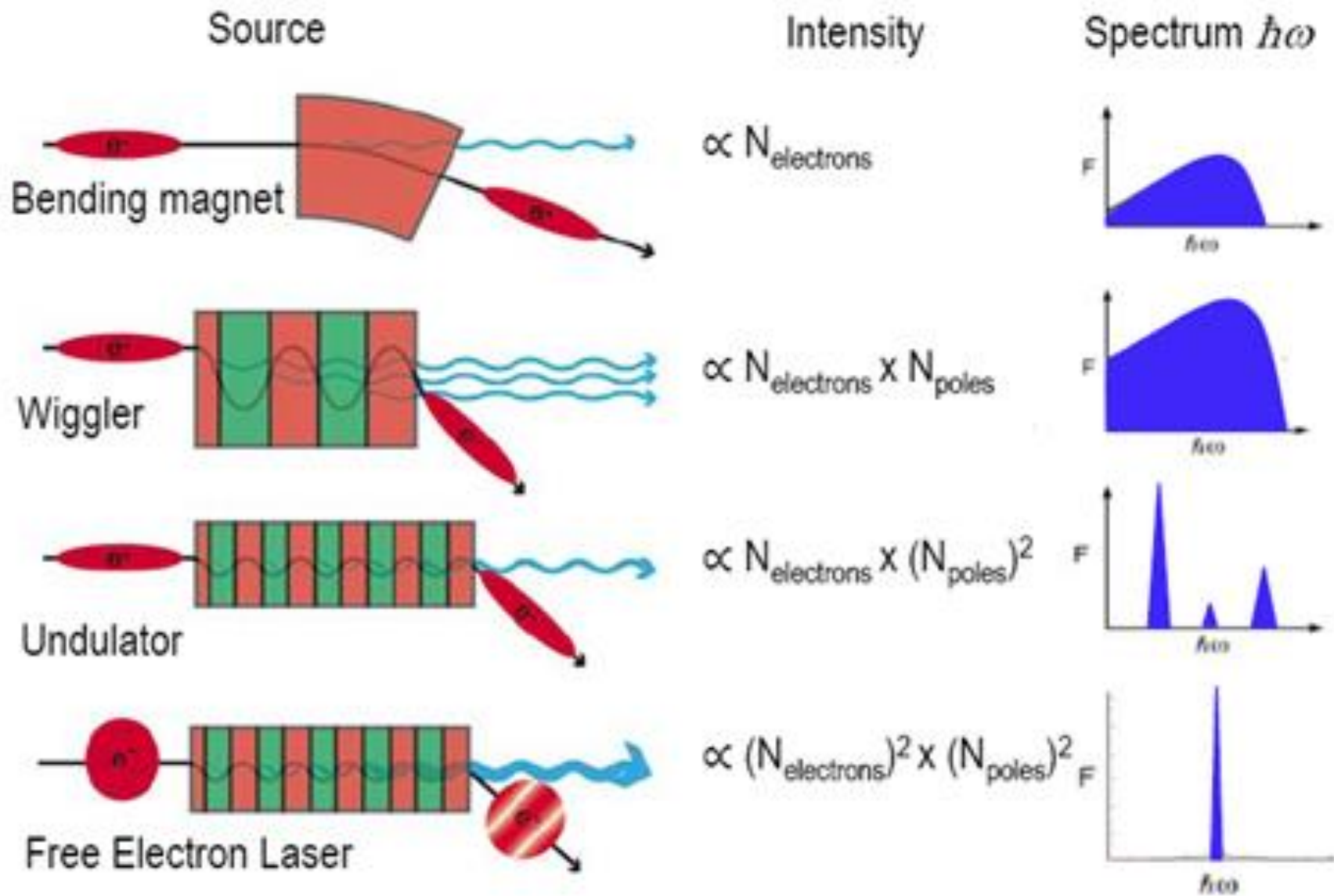
After an **electron** travels one undulator period  $\lambda_u$  of the **sinusoidal trajectory**, a plane wave (represented by alternating vertical arrows) overtakes the electron by one resonant wavelength  $\lambda_1$ . Thus, the undulator radiation carrying this resonant wavelength can *exchange energy* with the electron over many undulator periods.

Depending on the phase between the electrons and the plane wave, some electrons **gain** energy from the radiation, while other **lose** energy to the radiation. Slower electrons are accelerated while faster ones are slowed down leading to “**microbunching**”

# FEL microbunching

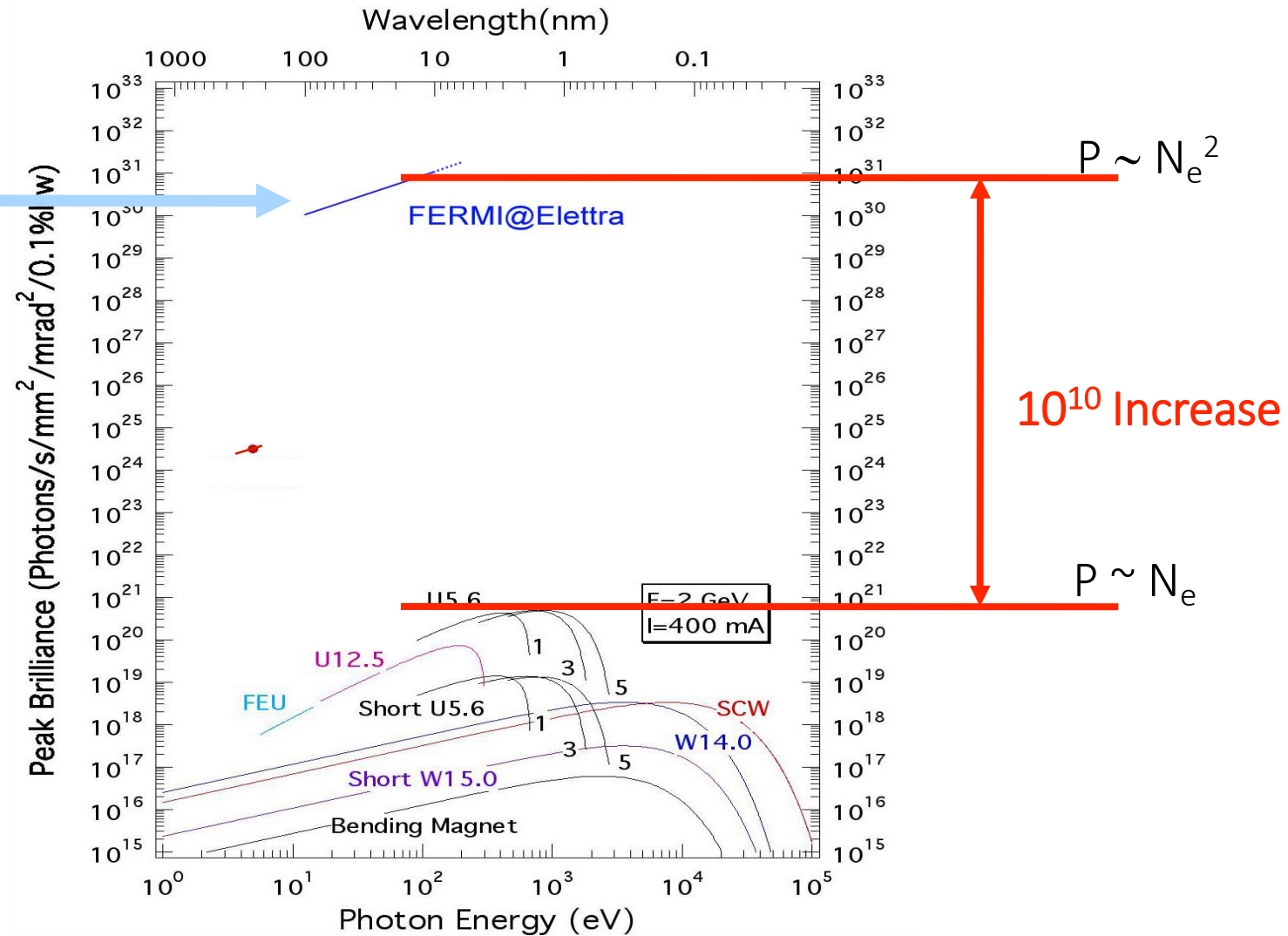
Growth of the radiation power and the electron beam microbunching as a function of the undulator distance for a high-gain FEL.



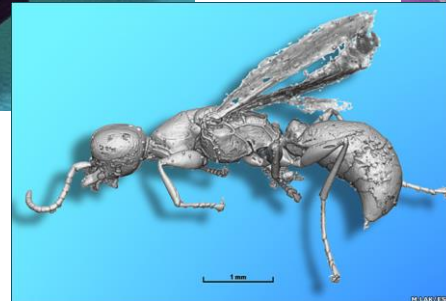
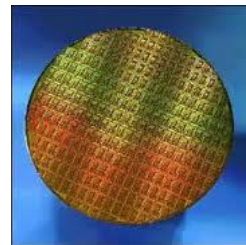
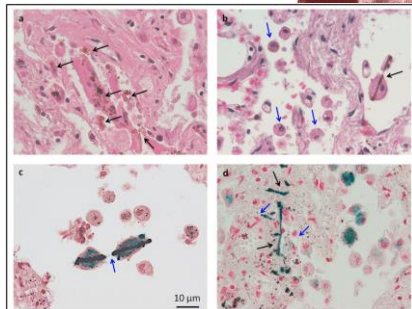
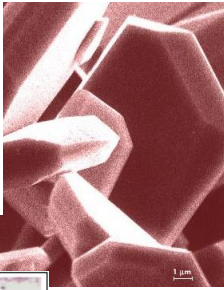
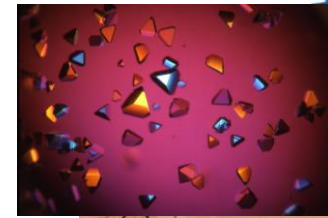
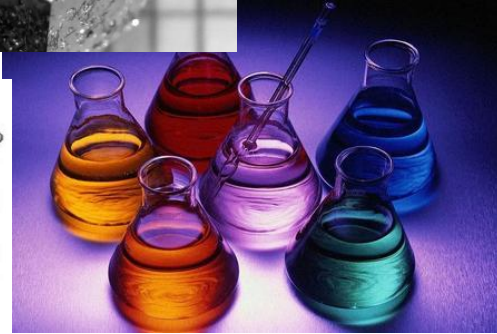
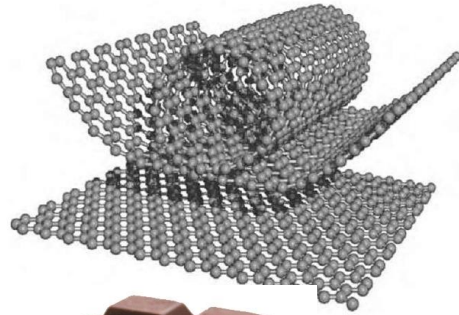
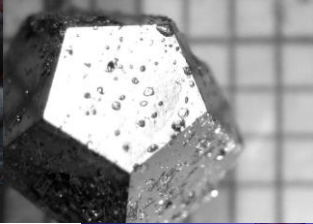


# FERMI brightness

FERMI@Elettra FEL









# At synchrotrons

High **spatial resolution**, high **sensitivity** and **element-specific information** not achievable at standard laboratory equipment

„.....In one trip to the synchrotron we collected more and better data in three days than in the previous ten years. I shut down all three X-ray spectrometers in the Boeing laboratory. A new era had arrived!....” – **Farrel Lytle**  
*J. Synchrotron Rad.* 6, 123-134 (1999).