

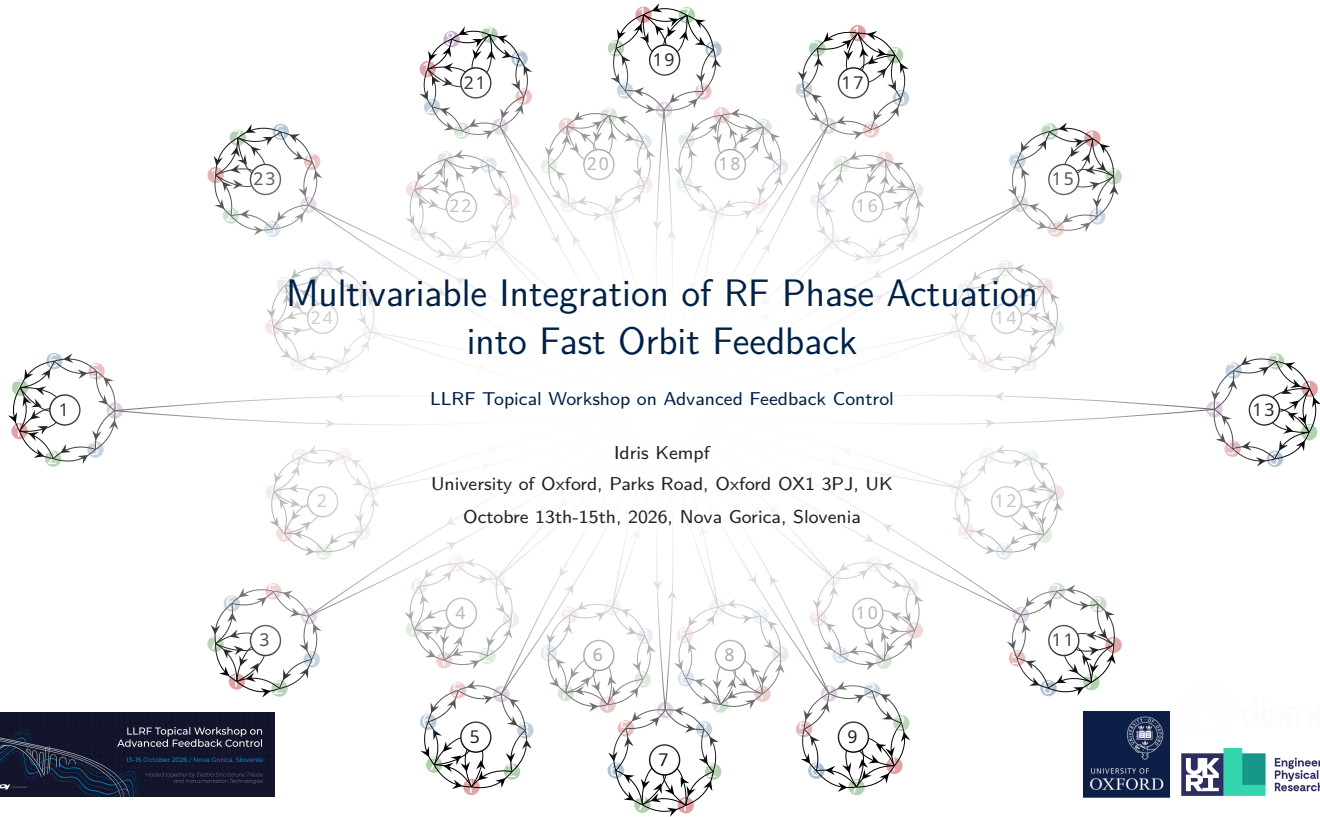
Multivariable Integration of RF Phase Actuation into Fast Orbit Feedback

LLRF Topical Workshop on Advanced Feedback Control

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October 13th-15th, 2026, Nova Gorica, Slovenia



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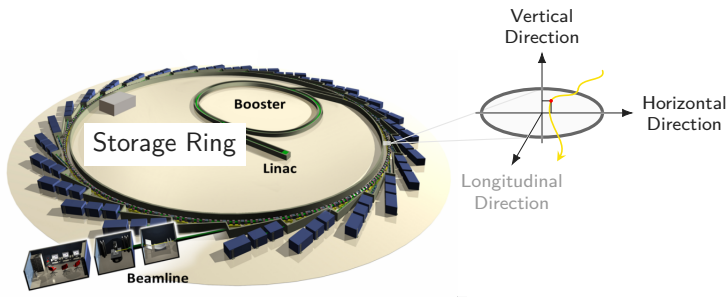
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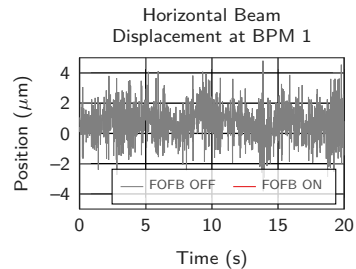


Fast Orbit Feedback (FOFB) in the Diamond-II Storage Ring



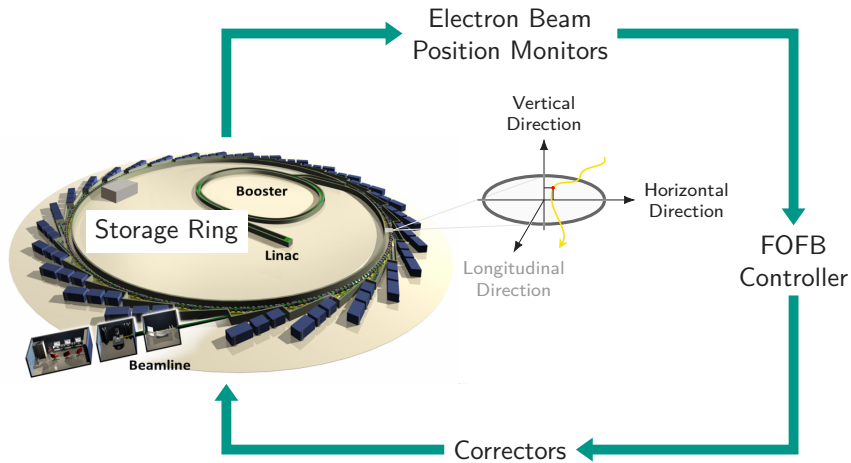
Short-term disturbances:

- Ground/girder vibrations
- RF/power supply noise
- Water cooling
- ...



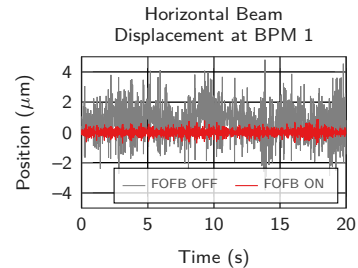
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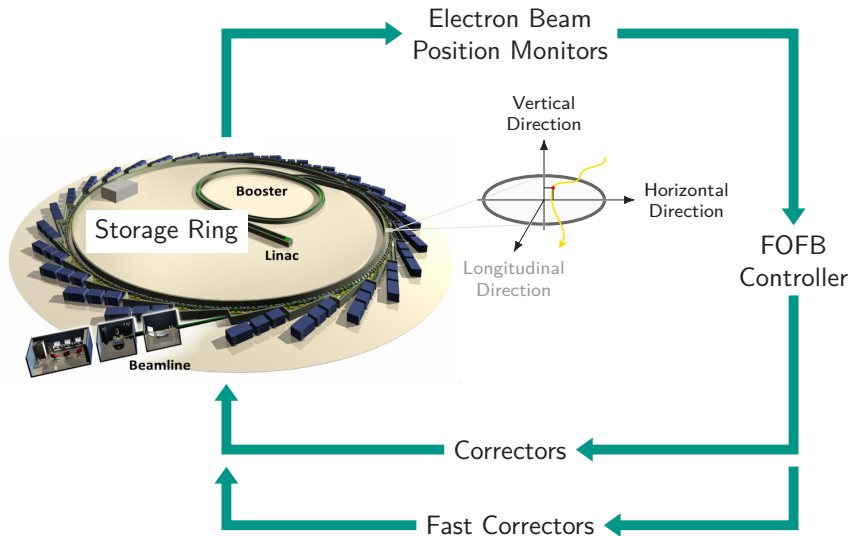
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⚡ Multi-Input Multi-Output ⚡

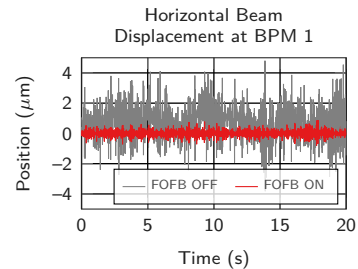
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Diamond-II Upgrade requires Enhanced FOFB Performance

1. Major Changes:

- New MBA lattice → smaller beam
- New beamlines → higher frequency resolution
- New insertion devices with upstream & downstream BPMs

- ⇒ M. G. Abbott *et al.*, “Diamond-II technical design report,” Aug. 2022.
- ⇒ L. T. Stant *et al.*, “Diamond-II electron beam position monitor development,” IBIC 2022.

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2. Increased Stability Requirements:

Parameter	Diamond	Diamond-II
Beam size H/V	123 μm 3.5 μm	30 μm 4 μm
Rel. stability	10 % at 100 Hz	3 % at 1 kHz
Abs. stability H/V	12 μm 0.35 μm	0.9 μm 0.12 μm

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3. Enhanced FOFB Specs:

Parameter	Diamond	Diamond-II
Closed-loop BW	140 Hz	≥ 1 kHz
Number of BPMs	173	252
Number of correctors	172	252 slow 144 fast
Sample frequency	10 kHz	100 kHz

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Sample frequency	10 kHz	100 kHz

Additional fast correctors require re-development of FOFB control methodology:

- ✓ Kempf et al., "Control of Multi-Array Cross-Directional Systems using the Generalised Singular Value Decomposition" *Automatica*, 2025.
- ✓ Kempf et al., "Fast Orbit Feedback for Diamond-II" *Proc. IBIC'23*, 2023.

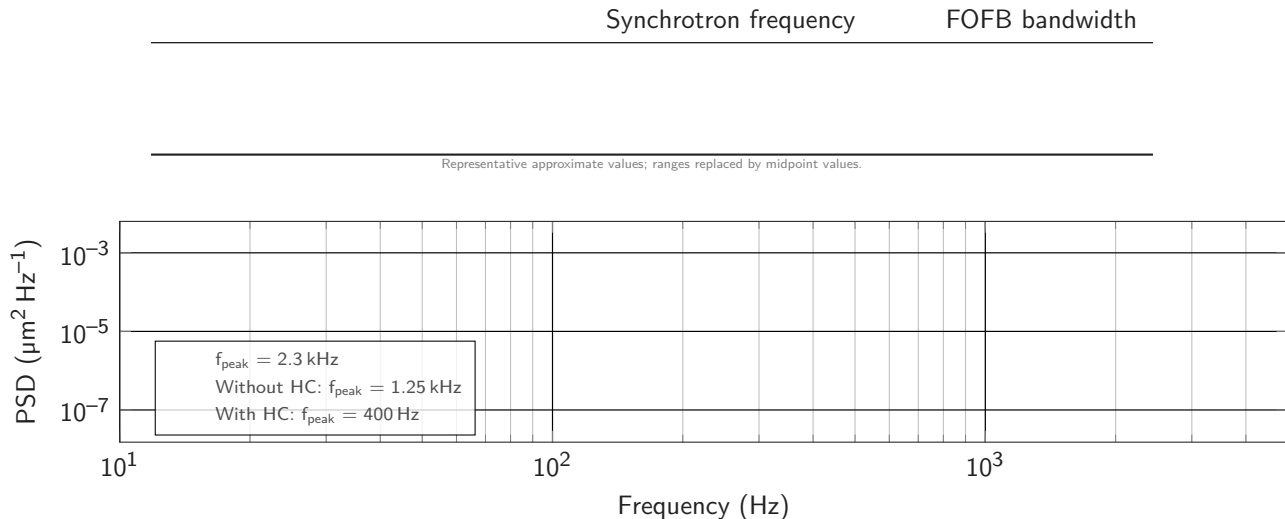
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Synchrotron Motion Disturbance

1. Introduction
2. Synchrotron Motion Disturbance
3. Control Architectures
4. Multi-Array Control using the GSVD
5. Preliminary Analysis and Simulation Results

New Disturbance: Synchrotron Motion Enters the FOFB Bandwidth at Diamond-II

💡 Energy oscillations at f_s appear as horizontal orbit motion through the lattice dispersion. 💡



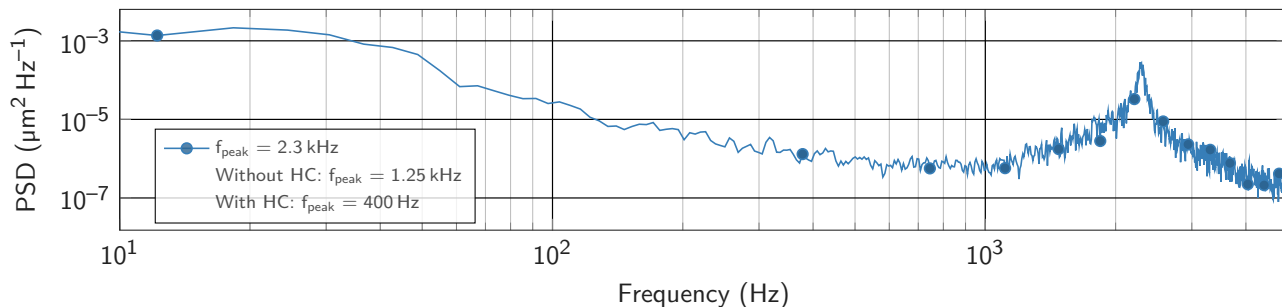
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	Synchrotron frequency		FOFB bandwidth
Diamond	2.3 kHz	$\gg$	200 Hz

Representative approximate values; ranges replaced by midpoint values.



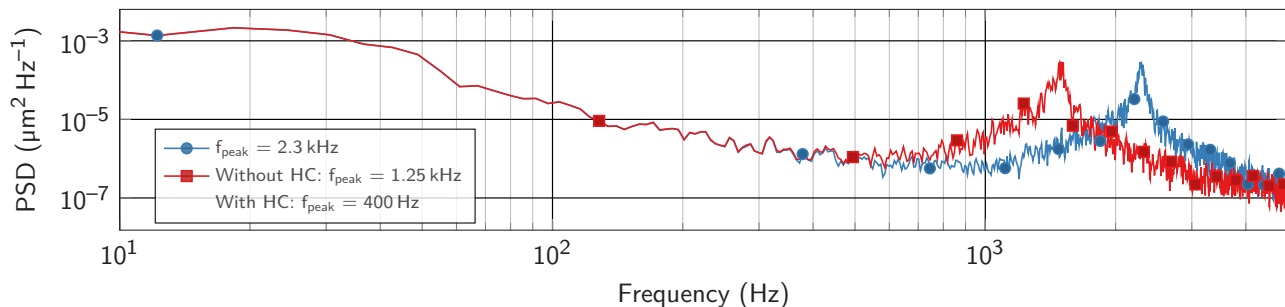
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	Synchrotron frequency		FOFB bandwidth
Diamond	2.3 kHz	$\gg$	200 Hz
Diamond-II bare lattice	1.25 kHz	$\approx$	1 kHz

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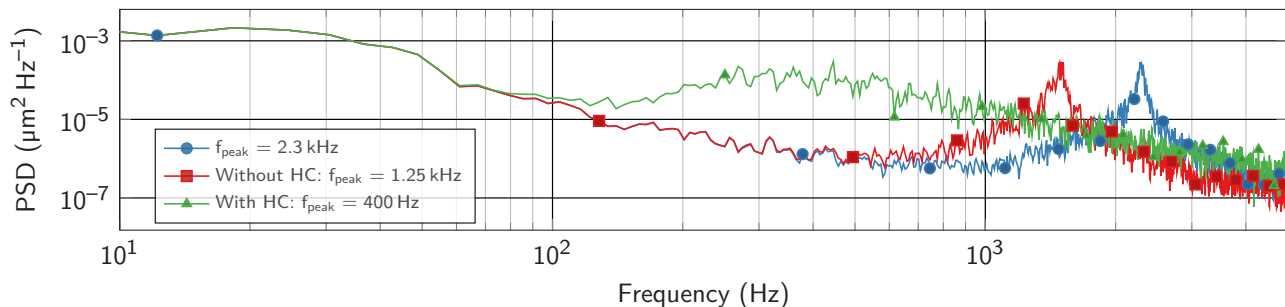
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Diamond	2.3 kHz	$\gg$	200 Hz
Diamond-II bare lattice	1.25 kHz	$\approx$	1 kHz
Diamond-II with 3 rd -harmonic cavity	0.1 kHz to 1 kHz	$<$	1 kHz

Representative approximate values; ranges replaced by midpoint values.

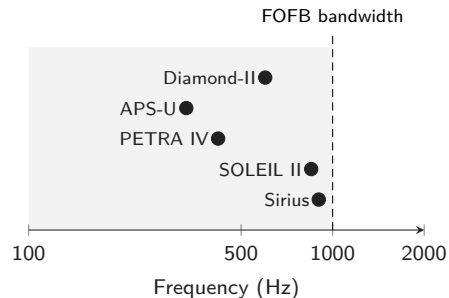


➡ M. G. Abbott *et al.*, "Diamond-II technical design report," Aug. 2022.

A General Phenomenon at Next-Generation Synchrotron Light Sources

	Synchrotron Frequency		FOFB Bandwidth
Diamond-II	600 Hz	<	1 kHz
APS-U ¹	330 Hz	<	1 kHz
PETRA IV ^{2,3}	420 Hz	<	1 kHz
SOLEIL II ^{4,5}	850 Hz	<	1 kHz
Sirius ^{6,7}	< 1 kHz	<	1 kHz

Representative approximate values illustrating harmonic-cavity operation; exact values depend on RF configuration.



⇒ ¹Kallakuri et al., *Phys. Rev. Accel. Beams*, 25, 082801, 2022.

⇒ ²Mirza et al., *Proc. IBIC 2022*, TUP43; ³Ebert, *DESY Technical Note 19-03*, 2019.

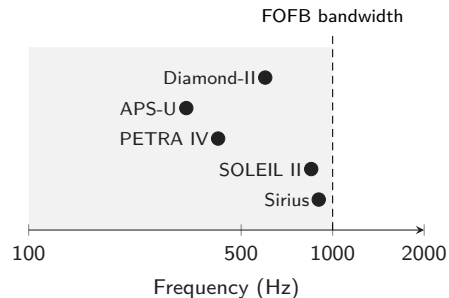
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⚡ Problem: FOFB not designed for “strong” & “high-frequency” synchrotron motion disturbances. ⚡

⇒ ¹Kallakuri et al., *Phys. Rev. Accel. Beams*, 25, 082801, 2022.

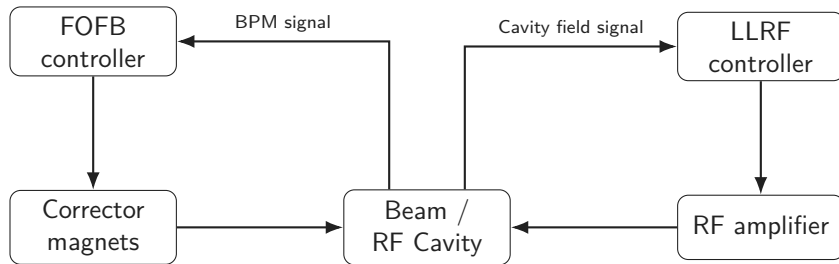
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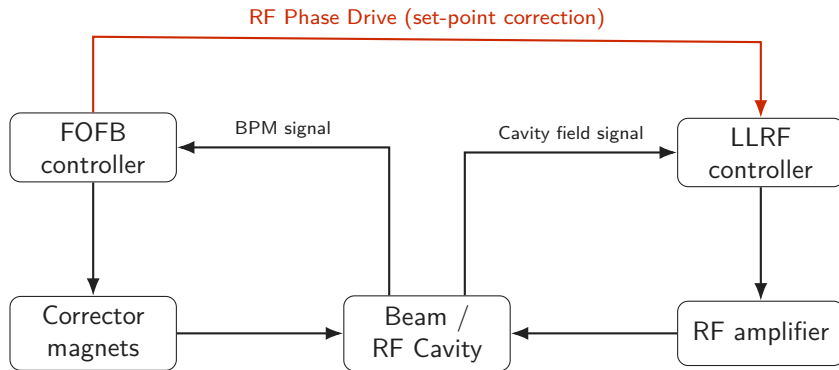
APS-U: Use the RF System as an Additional Actuator for Beam Stabilisation

- ✓ The same RF-beam coupling that creates the disturbance can be exploited for correction¹. ✓



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- ✓ The same RF-beam coupling that creates the disturbance can be exploited for correction¹. ✓



How can the RF System extend the FOFB?

Linear, multi-input multi-output orbit dynamics:

$$\mathbf{y}(j\omega) = \mathbf{R} g(j\omega) \mathbf{u}(j\omega) + \mathbf{d}(j\omega)$$

$\mathbf{y}(j\omega)$: Horiz. displacement in frequency domain; length n_{BPM}

$\mathbf{R}$: Orbit response matrix; size $n_{\text{BPM}} \times n_{\text{CM}}$

$g(j\omega)$: Scalar corrector dynamics

$\mathbf{u}(j\omega)$: Corrector inputs (currents); length n_{CM}

$\mathbf{d}(j\omega)$: Disturbance

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With RF System:

$$\mathbf{y}(j\omega) = \mathbf{R} g(j\omega) \mathbf{u}(j\omega) + \boldsymbol{\eta} g_{\text{RF}}(j\omega) \mathbf{u}_{\text{RF}}(j\omega) + \mathbf{d}(j\omega)$$

$\boldsymbol{\eta}$: Dispersion vector; length n_{BPM}

$g_{\text{RF}}(j\omega)$: Scalar RF dynamics

$\mathbf{u}_{\text{RF}}(j\omega)$: Corrector inputs (currents); length n_{CM}

Betatron and Synchrotron Motion are Nearly Orthogonal

Model for beam position $\mathbf{y} \stackrel{\text{def}}{=} (y_1, \dots, y_{n_{\text{BPM}}})^T$:

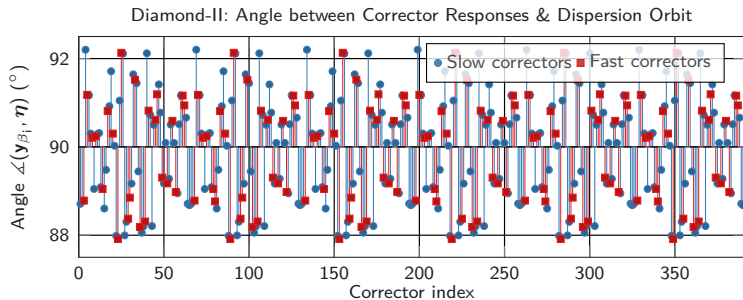
$$\mathbf{y} = \underbrace{\mathbf{y}_\beta}_{\text{Betatron motion}} + \underbrace{\mathbf{y}_\eta}_{\text{Synchrotron motion}} = \underbrace{\mathbf{y}_{\beta_1} + \dots + \mathbf{y}_{\beta_{n_{\text{CM}}}}}_{\text{Contribution by CMs}} + \underbrace{\mathbf{y}_\eta}_{\parallel \text{ to dispersion vector } \boldsymbol{\eta}}$$

Perfect decoupling:

$$\mathbf{y}_{\beta_i} \perp \boldsymbol{\eta} \quad \forall i = 1, \dots, n_{\text{CM}}$$



✓ FOFB/LLRF Loops decoupled ✓



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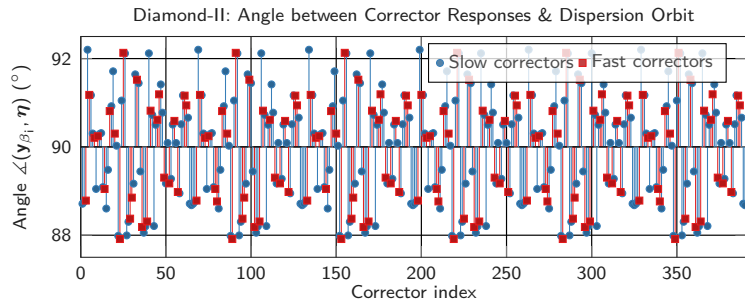
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$\Downarrow$

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? How should $\boldsymbol{\eta} \mathbf{g}_{\text{RF}}(j\omega) \mathbf{u}_{\text{RF}}(j\omega)$ be integrated when $\boldsymbol{\eta}^T \mathbf{R} \neq 0$?

Control Architectures

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5. Preliminary Analysis and Simulation Results

Selection of Control Architecture Candidates

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1.) Parallel feedback control of FOFB and RF phase actuation¹

- ▶ Design one controller for $\mathbf{u}(j\omega)$ and one for $\mathbf{u}_{\text{RF}}(j\omega)$ using $\mathbf{y}(j\omega)$ as feedback signal
- ▶ Each design ignores the other controller

⇒ ¹P. Kallakuri, A. Brill, J. Carwardine, and N. Sereno, Phys. Rev. Accel. Beams **25**, 082801 (2022)

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2.) Model-based one-way decoupling of FOFB and RF phase actuation

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3.) Multivariable integration of RF phase actuation into FOFB

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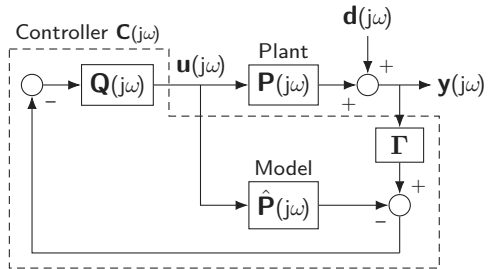
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Assumption: $\mathbf{R} \in \mathbb{R}^{n_{\text{BPM}} \times n_{\text{CM}}}$ spans the BPM space, i.e. $\text{rank}(\mathbf{R}) = n_{\text{BPM}}$.

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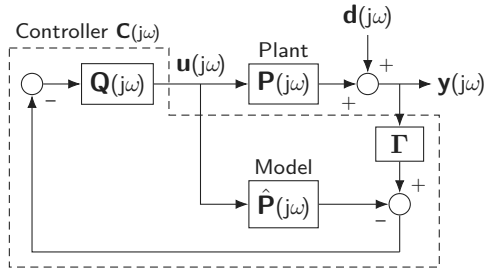
Basic Control Scheme for all Architectures Considered

Internal Model Control (IMC):

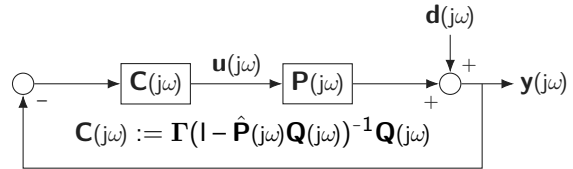


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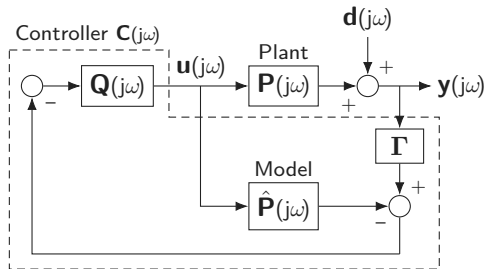


Equivalent Negative-Feedback Structure:

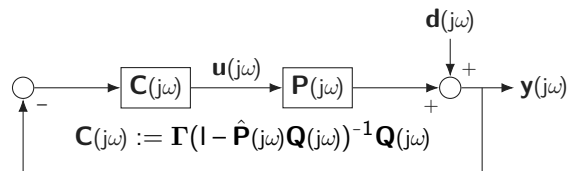


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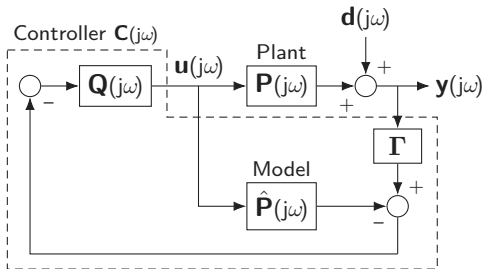
Example recipe:

Plant $P(j\omega)$	Model $\hat{P}(j\omega)$	IMC Filter $Q(j\omega)$	Regularisation Γ
$P(j\omega) = R g(j\omega)$	$\hat{P}(j\omega) = \hat{R} \hat{g}(j\omega)$	$Q(j\omega) = \hat{P}^{-1}(j\omega)T(j\omega)$	$\hat{R}\hat{R}^T(\hat{R}\hat{R}^T + \mu I)^{-1}$

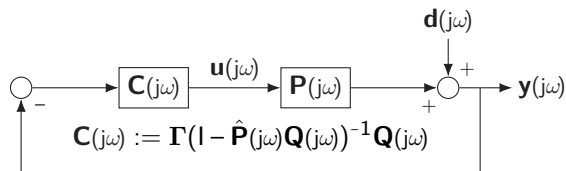
$\mu \geq 0$: scalar regularisation parameter

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$\mu \geq 0$: scalar regularisation parameter

► Results in **closed loop** ($P(j\omega) \equiv \hat{P}(j\omega)$ and $\Gamma = I$):

$$y(j\omega) = (1 - T(j\omega))d(j\omega) =: S(j\omega)d(j\omega)$$

► $T(j\omega)$ is the desired closed loop, and Γ attenuates low-gain directions

⇒ M. Morari and E. Zafiriou, *Robust Process Control*, 1989.

Parallel Feedback Control of FOFB and RF Phase Actuation

Correctors/FOFB

RF Phase Actuation

Model

$$\hat{\mathbf{P}}(j\omega) = \hat{\mathbf{R}} \hat{\mathbf{g}}(j\omega)$$

$$\hat{\mathbf{P}}_{\text{RF}}(j\omega) = \hat{\boldsymbol{\eta}} \hat{\mathbf{g}}_{\text{RF}}(j\omega)$$

Parallel Feedback Control of FOFB and RF Phase Actuation

	Correctors/FOFB	RF Phase Actuation
Model	$\hat{\mathbf{P}}(j\omega) = \hat{\mathbf{R}} \hat{\mathbf{g}}(j\omega)$	$\hat{\mathbf{P}}_{\text{RF}}(j\omega) = \hat{\boldsymbol{\eta}} \hat{\mathbf{g}}_{\text{RF}}(j\omega)$
IMC Filter	$\mathbf{Q}(j\omega) = \hat{\mathbf{R}}^{-1} \frac{\mathbf{T}(j\omega)}{\mathbf{g}(j\omega)}$	$\mathbf{Q}_{\text{RF}}(j\omega) = \frac{\hat{\boldsymbol{\eta}}^{\text{T}}}{\hat{\boldsymbol{\eta}}^{\text{T}} \hat{\boldsymbol{\eta}}} \frac{\mathbf{T}(j\omega)}{\mathbf{g}_{\text{RF}}(j\omega)}$

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Model	$\hat{\mathbf{P}}(j\omega) = \hat{\mathbf{R}} \hat{g}(j\omega)$	$\hat{\mathbf{P}}_{\text{RF}}(j\omega) = \hat{\boldsymbol{\eta}} \hat{g}_{\text{RF}}(j\omega)$
IMC Filter	$\mathbf{Q}(j\omega) = \hat{\mathbf{R}}^{-1} \frac{T(j\omega)}{g(j\omega)}$	$\mathbf{Q}_{\text{RF}}(j\omega) = \frac{\hat{\boldsymbol{\eta}}^T}{\hat{\boldsymbol{\eta}}^T \hat{\boldsymbol{\eta}}} \frac{T(j\omega)}{g_{\text{RF}}(j\omega)}$
Regularisation	$\boldsymbol{\Gamma} = \hat{\mathbf{R}} \hat{\mathbf{R}}^T (\hat{\mathbf{R}} \hat{\mathbf{R}}^T + \mu \mathbf{I})^{-1}$	$\boldsymbol{\Gamma}_{\text{RF}} = \hat{\boldsymbol{\eta}} \hat{\boldsymbol{\eta}}^T (\hat{\boldsymbol{\eta}} \hat{\boldsymbol{\eta}}^T + \mu \mathbf{I})^{-1}$

Parallel Feedback Control of FOFB and RF Phase Actuation

	Correctors/FOFB	RF Phase Actuation
Model	$\hat{\mathbf{P}}(j\omega) = \hat{\mathbf{R}} \hat{\mathbf{g}}(j\omega)$	$\hat{\mathbf{P}}_{\text{RF}}(j\omega) = \hat{\boldsymbol{\eta}} \hat{\mathbf{g}}_{\text{RF}}(j\omega)$
IMC Filter	$\mathbf{Q}(j\omega) = \hat{\mathbf{R}}^{-1} \frac{\mathbf{T}(j\omega)}{\hat{\mathbf{g}}(j\omega)}$	$\mathbf{Q}_{\text{RF}}(j\omega) = \frac{\hat{\boldsymbol{\eta}}^{\text{T}}}{\hat{\boldsymbol{\eta}}^{\text{T}} \hat{\boldsymbol{\eta}}} \frac{\mathbf{T}(j\omega)}{\hat{\mathbf{g}}_{\text{RF}}(j\omega)}$
Regularisation	$\boldsymbol{\Gamma} = \hat{\mathbf{R}} \hat{\mathbf{R}}^{\text{T}} (\hat{\mathbf{R}} \hat{\mathbf{R}}^{\text{T}} + \mu \mathbf{I})^{-1}$	$\boldsymbol{\Gamma}_{\text{RF}} = \hat{\boldsymbol{\eta}} \hat{\boldsymbol{\eta}}^{\text{T}} (\hat{\boldsymbol{\eta}} \hat{\boldsymbol{\eta}}^{\text{T}} + \mu \mathbf{I})^{-1}$
Neg.-FB Controller	$\mathbf{C}(j\omega) = \frac{1}{1-\mathbf{T}(j\omega)} \mathbf{Q}(j\omega) \boldsymbol{\Gamma}$	$\mathbf{C}_{\text{RF}}(j\omega) = \frac{1}{1-\mathbf{T}(j\omega)} \mathbf{Q}_{\text{RF}}(j\omega) \boldsymbol{\Gamma}_{\text{RF}}$

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IMC Filter	$\mathbf{Q}(j\omega) = \hat{\mathbf{R}}^{-1} \frac{\mathbf{T}(j\omega)}{\hat{\mathbf{g}}(j\omega)}$	$\mathbf{Q}_{\text{RF}}(j\omega) = \frac{\hat{\boldsymbol{\eta}}^T}{\hat{\boldsymbol{\eta}}^T \hat{\boldsymbol{\eta}}} \frac{\mathbf{T}(j\omega)}{\hat{\mathbf{g}}_{\text{RF}}(j\omega)}$
Regularisation	$\boldsymbol{\Gamma} = \hat{\mathbf{R}} \hat{\mathbf{R}}^T (\hat{\mathbf{R}} \hat{\mathbf{R}}^T + \mu \mathbf{I})^{-1}$	$\boldsymbol{\Gamma}_{\text{RF}} = \hat{\boldsymbol{\eta}} \hat{\boldsymbol{\eta}}^T (\hat{\boldsymbol{\eta}} \hat{\boldsymbol{\eta}}^T + \mu \mathbf{I})^{-1}$
Neg.-FB Controller	$\mathbf{C}(j\omega) = \frac{1}{1-\mathbf{T}(j\omega)} \mathbf{Q}(j\omega) \boldsymbol{\Gamma}$	$\mathbf{C}_{\text{RF}}(j\omega) = \frac{1}{1-\mathbf{T}(j\omega)} \mathbf{Q}_{\text{RF}}(j\omega) \boldsymbol{\Gamma}_{\text{RF}}$

Open Loop (Plant $\times$ Neg.-FB Controller) for accurate models:

$$\mathbf{L}(j\omega) = \boldsymbol{\Gamma} \frac{\mathbf{T}(j\omega)}{1-\mathbf{T}(j\omega)} + \boldsymbol{\Gamma}_{\text{RF}} \frac{\mathbf{T}(j\omega)}{1-\mathbf{T}(j\omega)}$$

Parallel Feedback Control of FOFB and RF Phase Actuation

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Disturbance-to-output sensitivity:

$$\mathbf{S}(j\omega) = (\mathbf{I} + \mathbf{L}(j\omega))^{-1} = \left(\mathbf{I} + \frac{\mathbf{T}(j\omega)}{1-\mathbf{T}(j\omega)} (\mathbf{\Gamma} + \mathbf{\Gamma}_{\text{RF}}) \right)^{-1}$$

Parallel Feedback Control of FOFB and RF Phase Actuation

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Parallel feedback: Double loop gain in $\hat{\boldsymbol{\eta}}$ direction.

Model-Based One-Way Decoupling of FOFB and RF Phase Actuation

1. Close the RF loop using the controller from the previous slide:

$$\mathbf{u}_{\text{RF}}(j\omega) = -\mathbf{C}_{\text{RF}}(j\omega) \mathbf{y}(j\omega).$$

Model-Based One-Way Decoupling of FOFB and RF Phase Actuation

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2. Redefine the output signal for the correctors:

$$\bar{\mathbf{y}}(j\omega) = \mathbf{y}(j\omega) - \hat{\mathbf{P}}_{\text{RF}}(j\omega) \mathbf{u}_{\text{RF}}(j\omega).$$

For accurate models: $\bar{\mathbf{y}}(j\omega) = \mathbf{P}(j\omega) \mathbf{u}(j\omega) + \mathbf{d}(j\omega)$.

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3. Apply the original FOFB controller to the reconstructed output:

$$\mathbf{u}(j\omega) = -\mathbf{C}(j\omega) \bar{\mathbf{y}}(j\omega).$$

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$$\mathbf{S}(j\omega) = \left(\mathbf{I} + \mathbf{\Gamma}_{\text{RF}} \frac{\mathbf{T}(j\omega)}{1 - \mathbf{T}(j\omega)} \right)^{-1} \left(\mathbf{I} + \mathbf{\Gamma} \frac{\mathbf{T}(j\omega)}{1 - \mathbf{T}(j\omega)} \right)^{-1}.$$

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One-way decoupling: Double sensitivity slope along $\hat{\eta}$ for $\mathbf{\Gamma} = \mathbf{\Gamma}_{\text{RF}} = \mathbf{I}$: $(1 - \mathbf{T}(j\omega))^2$ instead of $1 - \mathbf{T}(j\omega)$.

Multi-Array Control using the GSVD

1. Introduction
2. Synchrotron Motion Disturbance
3. Control Architectures
4. Multi-Array Control using the GSVD
5. Preliminary Analysis and Simulation Results

Modal Decoupling for Single-Array Systems (one corrector type)

1) Linear, multi-input multi-output orbit dynamics:

$$\mathbf{y}(j\omega) = \mathbf{R} g(j\omega) \mathbf{u}(j\omega) + \mathbf{d}(j\omega)$$

$\mathbf{y}(j\omega)$: Horiz. displacement in frequency domain; length n_{BPM}

$\mathbf{R}$: Orbit response matrix; size $n_{\text{BPM}} \times n_{\text{CM}}$

$g(j\omega)$: Scalar corrector dynamics

$\mathbf{u}(j\omega)$: Corrector inputs (currents); length n_{CM}

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2) Substitute thin singular value decomposition of $\mathbf{R}$:

$$\mathbf{y}(j\omega) = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^T g(j\omega) \mathbf{u}(j\omega) + \mathbf{d}(j\omega)$$

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3) Left-multiply with $\mathbf{U}^T$ and swap $g(j\omega)$ and $\mathbf{V}^T$:

$$\mathbf{U}^T \mathbf{y}(j\omega) = \mathbf{U}^T \mathbf{U} \overset{\text{I}}{\Sigma} g(j\omega) \mathbf{V}^T \mathbf{u}(j\omega) + \mathbf{U}^T \mathbf{d}(j\omega)$$

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4) Define modal variables $\tilde{\mathbf{y}}(j\omega) \stackrel{\text{def}}{=} \mathbf{U}^T \mathbf{y}(j\omega)$, $\tilde{\mathbf{u}}(j\omega) \stackrel{\text{def}}{=} \mathbf{V}^T \mathbf{u}(j\omega)$, and $\tilde{\mathbf{d}}(j\omega) \stackrel{\text{def}}{=} \mathbf{U}^T \mathbf{d}(j\omega)$:

$$\tilde{\mathbf{y}}(j\omega) = \Sigma \mathbf{g}(j\omega) \tilde{\mathbf{u}}(j\omega) + \tilde{\mathbf{d}}(j\omega), \quad \text{where } \Sigma = \text{diag}(\sigma_1, \dots, \sigma_{n_{\text{BPM}}}) \succ 0.$$

✓ Multi-Input Multi-Output System → Decoupled Single-Input Single-Output Systems ✓

Modal Decoupling for Single-Array Systems (one corrector type)

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5) Design IMC filter as for previous approaches

⇒ S. Gayadeen and S.R. Duncan, "Design of an electron beam stabilisation controller for a synchrotron", *Control Engineering Practice*, 2014.

Input Balancing for Systems with Two “Arrays” (one corrector type + RF)

1) Orbit dynamics with dispersion contribution:

$$\mathbf{y}(j\omega) = \mathbf{R} \mathbf{g}(j\omega) \mathbf{u}(j\omega) + \boldsymbol{\eta} \mathbf{g}_{\text{RF}}(j\omega) \mathbf{u}_{\text{RF}}(j\omega) + \mathbf{d}(j\omega)$$

⇒ H. Kim et al., “Fast Orbit Feedback using the GSVD for Systems with Multiple Slow Corrector Arrays”, *Proc. ICALEPCS 2025*, THCG001.

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2) Define $\hat{\mathbf{u}}_{\text{RF}}(j\omega) = \frac{g_{\text{RF}}(j\omega)}{g(j\omega)} \mathbf{u}_{\text{RF}}(j\omega) \longleftrightarrow \mathbf{u}_{\text{RF}}(j\omega) = \frac{g(j\omega)}{g_{\text{RF}}(j\omega)} \hat{\mathbf{u}}_{\text{RF}}(j\omega)$, substitute, and rearrange:

$$\mathbf{y}(j\omega) = \underbrace{\begin{bmatrix} \mathbf{R} & \boldsymbol{\eta} \end{bmatrix}}_{\hat{\mathbf{R}}} g(j\omega) \underbrace{\begin{pmatrix} \mathbf{u}(j\omega) \\ \hat{\mathbf{u}}_{\text{RF}}(j\omega) \end{pmatrix}}_{\hat{\mathbf{u}}(j\omega)} + \mathbf{d}(j\omega) = \hat{\mathbf{R}} g(j\omega) \hat{\mathbf{u}}(j\omega) + \mathbf{d}(j\omega)$$

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3) Proceed with single-array methodology and $\mathbf{\Gamma}_{\text{joint}} = (\mathbf{R}\mathbf{R}^T + \boldsymbol{\eta}\boldsymbol{\eta}^T)(\mathbf{R}\mathbf{R}^T + \boldsymbol{\eta}\boldsymbol{\eta}^T + \mu\mathbf{I})^{-1} \dots$

⇒ H. Kim et al., “Fast Orbit Feedback using the GSVD for Systems with Multiple Slow Corrector Arrays”, *Proc. ICALEPCS 2025*, THCG001.

Generalised Singular Value Decomposition for Two Matrices

Given $\mathbf{R}_s$, size $n_{\text{BPM}} \times n_s$ with $n_s \equiv n_{\text{BPM}}$, and $\mathbf{R}_f$, size $n_{\text{BPM}} \times n_f$ with $n_f \leq n_{\text{BPM}}$:

$$\mathbf{R}_s = \mathbf{X} \begin{bmatrix} \Sigma_s & \\ & \mathbf{I} \end{bmatrix} \mathbf{V}_s^T \quad \mathbf{R}_f = \mathbf{X} \begin{bmatrix} \Sigma_f \\ 0 \end{bmatrix} \mathbf{V}_f^T$$

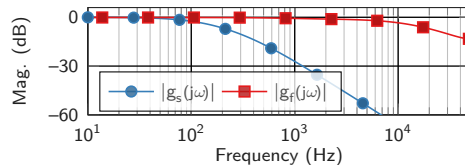
$\mathbf{X}$: always invertible but generally non-orthonormal ($\mathbf{X}^T \mathbf{X} \neq \mathbf{I}$); size $n_{\text{BPM}} \times n_{\text{BPM}}$
 $\Sigma_{(\cdot)}$: generalised singular values ($\neq$ singular values); size $n_f \times n_f$
 $\mathbf{V}_{(\cdot)}$: orthonormal ($\mathbf{V}_{(\cdot)}^T \mathbf{V}_{(\cdot)} = \mathbf{I}$); size $n_{(\cdot)} \times n_y$

⇒ G.H. Golub and C.F. Van Loan, *Matrix Computations*, 3rd ed., 2013.

Modal Decoupling for Two-Array Systems (slow & fast correctors)

1) Linear, multi-input multi-output orbit dynamics:

$$\mathbf{y}(j\omega) = \mathbf{R}_s \mathbf{g}_s(j\omega) \mathbf{u}_s(j\omega) + \mathbf{R}_f \mathbf{g}_f(j\omega) \mathbf{u}_f(j\omega) + \mathbf{d}(j\omega)$$

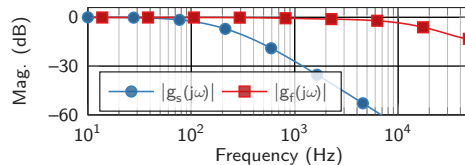


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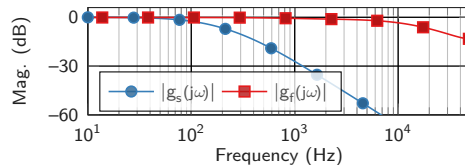
2) Substitute GSVD of $\mathbf{R}_s$ and $\mathbf{R}_f$...



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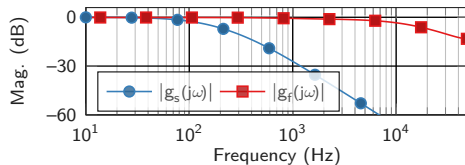
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3) Left-multiply with $\mathbf{X}^{-1}$ and swap $\mathbf{g}_{(\cdot)}(j\omega)$ and $\mathbf{V}_{(\cdot)}^T$...

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4) Define modal variables $\tilde{\mathbf{y}}(j\omega) \stackrel{\text{def}}{=} \mathbf{X}^{-1} \mathbf{y}(j\omega)$, $\tilde{\mathbf{u}}_{(\cdot)}(j\omega) \stackrel{\text{def}}{=} \mathbf{V}_{(\cdot)}^T \mathbf{u}_{(\cdot)}(j\omega)$, and $\tilde{\mathbf{d}}(j\omega) \stackrel{\text{def}}{=} \mathbf{X}^{-1} \mathbf{d}(j\omega)$:

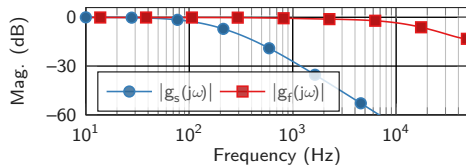
$$\tilde{\mathbf{y}}(j\omega) = \begin{bmatrix} \Sigma_s & \\ & \mathbf{I} \end{bmatrix} \mathbf{g}_s(j\omega) \tilde{\mathbf{u}}_s(j\omega) + \begin{bmatrix} \Sigma_f \\ 0 \end{bmatrix} \mathbf{g}_f(j\omega) \tilde{\mathbf{u}}_f(j\omega) + \tilde{\mathbf{d}}(j\omega)$$

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Modal Decoupling for Two-Array Systems (slow & fast correctors)

1) Linear, multi-input multi-output orbit dynamics:

$$\mathbf{y}(j\omega) = \mathbf{R}_s \mathbf{g}_s(j\omega) \mathbf{u}_s(j\omega) + \mathbf{R}_f \mathbf{g}_f(j\omega) \mathbf{u}_f(j\omega) + \mathbf{d}(j\omega)$$



2) Substitute GSVD of $\mathbf{R}_s$ and $\mathbf{R}_f$...

3) Left-multiply with $\mathbf{X}^{-1}$ and swap $\mathbf{g}_{(\cdot)}(j\omega)$ and $\mathbf{V}_{(\cdot)}^T$...

4) Define modal variables $\tilde{\mathbf{y}}(j\omega) \stackrel{\text{def}}{=} \mathbf{X}^{-1} \mathbf{y}(j\omega)$, $\tilde{\mathbf{u}}_{(\cdot)}(j\omega) \stackrel{\text{def}}{=} \mathbf{V}_{(\cdot)}^T \mathbf{u}_{(\cdot)}(j\omega)$, and $\tilde{\mathbf{d}}(j\omega) \stackrel{\text{def}}{=} \mathbf{X}^{-1} \mathbf{d}(j\omega)$:

$$\tilde{\mathbf{y}}(j\omega) = \begin{bmatrix} \Sigma_s & \\ & \mathbf{I} \end{bmatrix} \mathbf{g}_s(j\omega) \tilde{\mathbf{u}}_s(j\omega) + \begin{bmatrix} \Sigma_f \\ 0 \end{bmatrix} \mathbf{g}_f(j\omega) \tilde{\mathbf{u}}_f(j\omega) + \tilde{\mathbf{d}}(j\omega)$$

✓ Multi-Input Multi-Output System → Decoupled Two-Input Single-Output Systems ✓

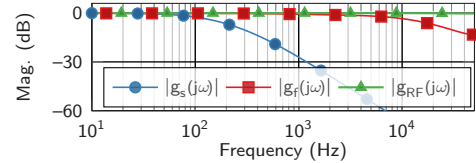
5) Design IMC using mid ranging to split effort between slow & fast correctors

⇒ I. Kempf et al., "Control of Multi-Array Cross-Directional Systems using the Generalised Singular Value Decomposition", *Automatica*, 2025.

Input Balancing for Systems with More than Two Arrays

1) Orbit dynamics with dispersion contribution:

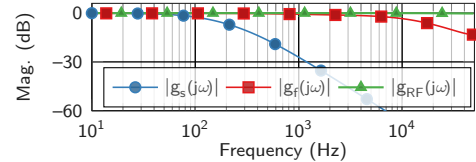
$$\mathbf{y}(j\omega) = \mathbf{R}_s \mathbf{g}_s(j\omega) \mathbf{u}_s(j\omega) + \mathbf{R}_f \mathbf{g}_f(j\omega) \mathbf{u}_f(j\omega) + \boldsymbol{\eta} \mathbf{g}_{RF}(j\omega) \mathbf{u}_{RF}(j\omega) + \mathbf{d}(j\omega)$$



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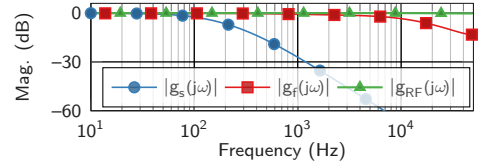


💡 RF amplifier bandwidth of 100 kHz $\gg$ Fast CM bandwidth of 10 kHz. 💡

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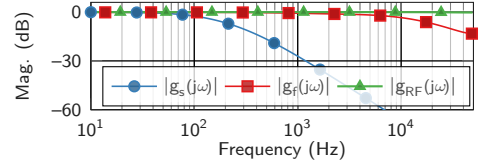
2) Define $\hat{\mathbf{u}}_{\text{RF}}(j\omega) = \frac{\mathbf{g}_{\text{RF}}(j\omega)}{\mathbf{g}_f(j\omega)} \mathbf{u}_{\text{RF}}(j\omega) \longleftrightarrow \mathbf{u}_{\text{RF}}(j\omega) = \frac{\mathbf{g}_f(j\omega)}{\mathbf{g}_{\text{RF}}(j\omega)} \hat{\mathbf{u}}_{\text{RF}}(j\omega)$, substitute, and rearrange:

$$\mathbf{y}(j\omega) = \mathbf{R}_s \mathbf{g}_s(j\omega) \mathbf{u}_s(j\omega) + \underbrace{\begin{bmatrix} \mathbf{R}_f & \boldsymbol{\eta} \end{bmatrix}}_{\hat{\mathbf{R}}_f} \mathbf{g}_f(j\omega) \underbrace{\begin{bmatrix} \mathbf{u}_f(j\omega) \\ \hat{\mathbf{u}}_{\text{RF}}(j\omega) \end{bmatrix}}_{\hat{\mathbf{u}}_f(j\omega)} + \mathbf{d}(j\omega)$$

Input Balancing for Systems with More than Two Arrays

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2) Define $\hat{\mathbf{u}}_{\text{RF}}(j\omega) = \frac{\mathbf{g}_{\text{RF}}(j\omega)}{\mathbf{g}_f(j\omega)} \mathbf{u}_{\text{RF}}(j\omega) \longleftrightarrow \mathbf{u}_{\text{RF}}(j\omega) = \frac{\mathbf{g}_f(j\omega)}{\mathbf{g}_{\text{RF}}(j\omega)} \hat{\mathbf{u}}_{\text{RF}}(j\omega)$, substitute, and rearrange:

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3) Proceed with two-array methodology...

⇒ H. Kim et al., "Fast Orbit Feedback using the GSVD for Systems with Multiple Slow Corrector Arrays", *Proc. ICALEPCS 2025*, THCG001.

Input-to-Output Sensitivity for Modal-Based Control

Single-array ($\mathbf{R}$) or single-array with input balancing ($\hat{\mathbf{R}} = [\mathbf{R} \quad \boldsymbol{\eta}]$):

$$\mathbf{S}(j\omega) = \left(\mathbf{I} + \frac{\mathbf{T}(j\omega)}{1 - \mathbf{T}(j\omega)} \boldsymbol{\Gamma} \right)^{-1} = \mathbf{U} \operatorname{diag} \left(\frac{1 - \mathbf{T}(j\omega)}{1 - \mathbf{T}(j\omega)(1 - \gamma_i)} \right)_{i=1}^{n_{\text{BPM}}} \mathbf{U}^T$$

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Two-array or two-array with input balancing:

$$\begin{aligned} \mathbf{S}(j\omega) &= \left[\mathbf{I} + \left(\frac{\mathbf{T}_{\text{TISO}}(j\omega)}{1 - \mathbf{T}_{\text{TISO}}(j\omega)} \boldsymbol{\Pi}_f + \frac{\mathbf{T}_{\text{SISO}}(j\omega)}{1 - \mathbf{T}_{\text{SISO}}(j\omega)} (\mathbf{I} - \boldsymbol{\Pi}_f) \right) \boldsymbol{\Gamma} \right]^{-1}, \\ &= \mathbf{U}_\Gamma \operatorname{diag} \left(\left\{ \frac{1 - \mathbf{T}_{\text{TISO}}(j\omega)}{1 - \mathbf{T}_{\text{TISO}}(j\omega) + \gamma_i \mathbf{T}_{\text{TISO}}(j\omega)} \right\}_{i=1}^{n_f}, \left\{ \frac{1 - \mathbf{T}_{\text{SISO}}(j\omega)}{1 - \mathbf{T}_{\text{SISO}}(j\omega) + \gamma_i \mathbf{T}_{\text{SISO}}(j\omega)} \right\}_{i=n_f+1}^{n_{\text{BPM}}} \right) \mathbf{U}_\Gamma^T. \end{aligned}$$

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Modal control: Uniform attenuation within each output subspace with mode-dependent attenuation.

Preliminary Analysis and Simulation Results

1. Introduction
2. Synchrotron Motion Disturbance
3. Control Architectures
4. Multi-Array Control using the GSVD
5. Preliminary Analysis and Simulation Results

RF Integration with one corrector type: Unregularised Sensitivity

For accurate models and a full-rank magnet array:

$$\mathbf{S}_{\text{joint}}(j\omega) = (1 - T(j\omega))\mathbf{I}$$

$$\mathbf{S}_{\text{parallel}}(j\omega) = (1 - T(j\omega)) \left(\mathbf{I} - \frac{T(j\omega)}{1 + T(j\omega)} \frac{\hat{\boldsymbol{\eta}}\hat{\boldsymbol{\eta}}^T}{\|\hat{\boldsymbol{\eta}}\|^2} \right)$$

$$\mathbf{S}_{\text{one-way}}(j\omega) = (1 - T(j\omega)) \left(\mathbf{I} - T(j\omega) \frac{\hat{\boldsymbol{\eta}}\hat{\boldsymbol{\eta}}^T}{\|\hat{\boldsymbol{\eta}}\|^2} \right)$$

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Along $\hat{\boldsymbol{\eta}}$ direction:

$$S_{\text{joint}}(j\omega)|_{\hat{\boldsymbol{\eta}}} = 1 - T(j\omega)$$

$$S_{\text{parallel}}(j\omega)|_{\hat{\boldsymbol{\eta}}} = \frac{1 - T(j\omega)}{1 + T(j\omega)}$$

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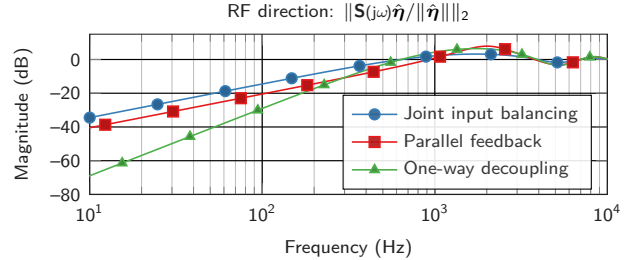
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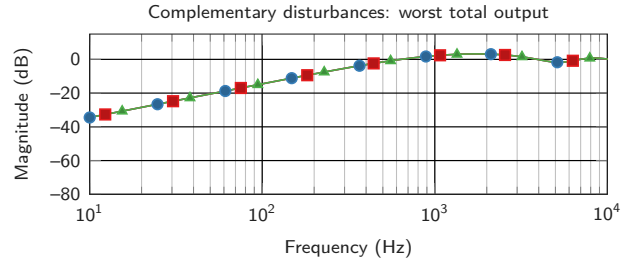
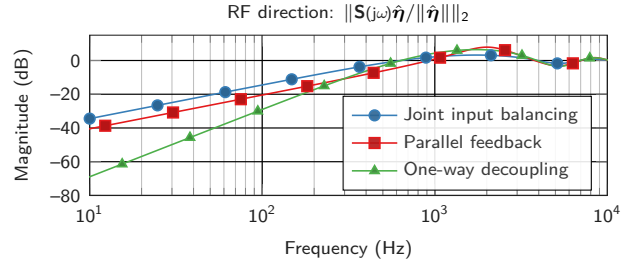
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$$\mathbf{S}_{\text{one-way}}(j\omega)|_{\hat{\boldsymbol{\eta}}} = (1 - T(j\omega))^2$$

Along a direction $\mathbf{d} \perp \hat{\boldsymbol{\eta}}$:

$$\mathbf{S}_{(\cdot)}(j\omega)|_{\mathbf{d} \perp \hat{\boldsymbol{\eta}}} = 1 - T(j\omega)$$



RF Integration: Unregularised Dispersion-Direction Robustness Margins

For accurate models and a full-rank magnet array:

$$L_{\eta,\text{joint}}(j\omega) = \frac{T(j\omega)}{1 - T(j\omega)},$$

$$L_{\eta,\text{parallel}}(j\omega) = \frac{2T(j\omega)}{1 - T(j\omega)},$$

$$L_{\eta,\text{one-way}}(j\omega) = \frac{1}{(1 - T(j\omega))^2} - 1.$$

RF Integration: Unregularised Dispersion-Direction Robustness Margins

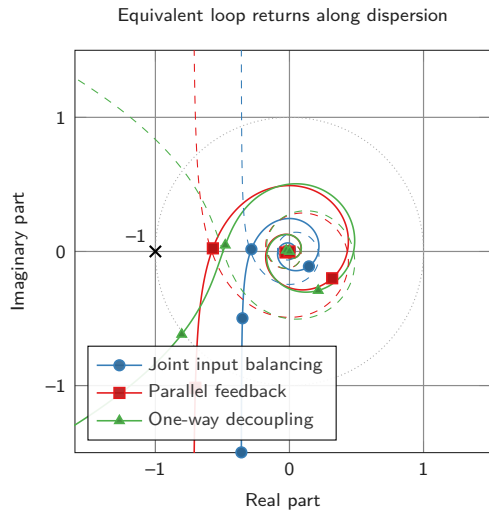
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Architecture	GM (dB)	PM (°)	$f_{ L =1}$ (Hz)
Joint input balancing	10.75	69.1	535
Parallel feedback	4.73	46.3	1111
One-way decoupling	6.09	37.4	939



Solid: $f > 0$; dashed: $f < 0$. Dotted circle: $|L| = 1$.

RF Integration with one corrector type: Regularised Sensitivity

⚡ Without regularisation: FOFB can be unstable¹⁻³. ⚡

⇒ ¹E. Plouviez, "Fast Positional Global Feedback for Storage Rings," DIPAC, 1999.

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$$\mathbf{S}_{\text{parallel}}(j\omega) = \left(\mathbf{I} + \frac{\mathbf{T}(j\omega)}{1 - \mathbf{T}(j\omega)} (\mathbf{\Gamma} + \mathbf{\Gamma}_{\text{RF}}) \right)^{-1}$$

$$\mathbf{S}_{\text{one-way}}(j\omega) = \left(\mathbf{I} + \frac{\mathbf{T}(j\omega)}{1 - \mathbf{T}(j\omega)} \mathbf{\Gamma}_{\text{RF}} \right)^{-1} \left(\mathbf{I} + \frac{\mathbf{T}(j\omega)}{1 - \mathbf{T}(j\omega)} \mathbf{\Gamma} \right)^{-1}$$

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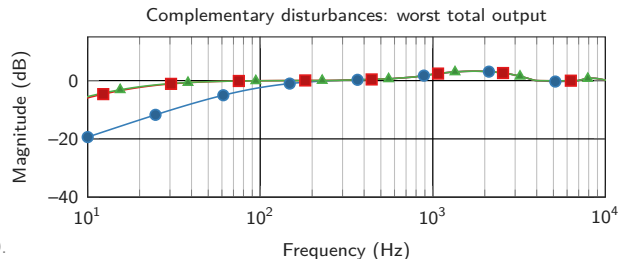
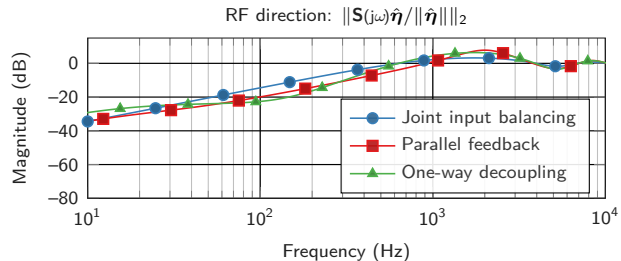
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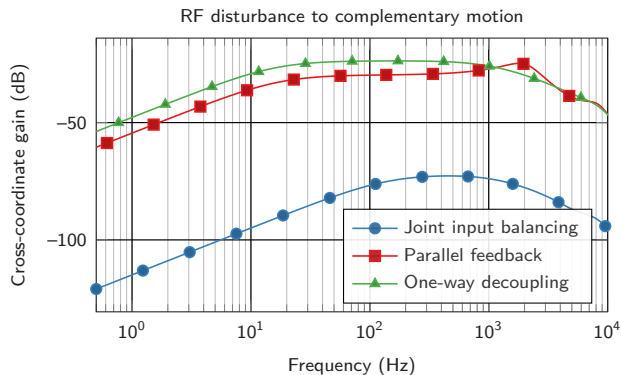
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RF Integration with one corrector type: Regularisation-Induced Coupling

RF disturbance $\rightarrow$ complementary directions:

$$\left\| \left(I - \frac{\hat{\eta} \hat{\eta}^T}{\|\hat{\eta}\|_2^2} \right) \mathbf{S}_{(j\omega)} \frac{\hat{\eta}}{\|\hat{\eta}\|_2} \right\|_2$$



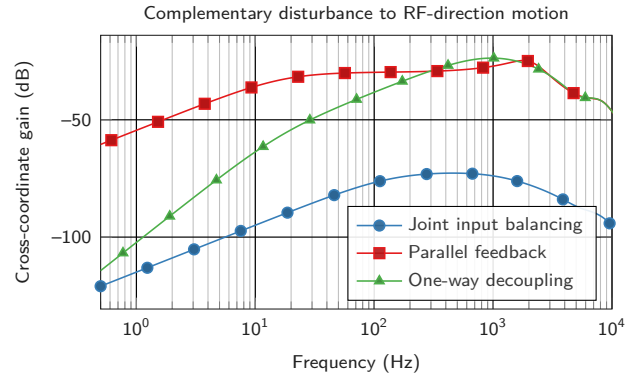
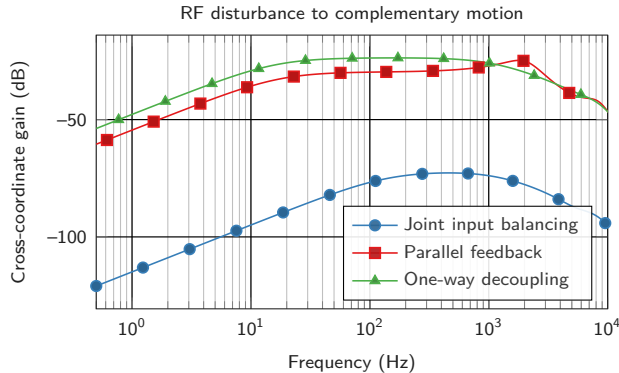
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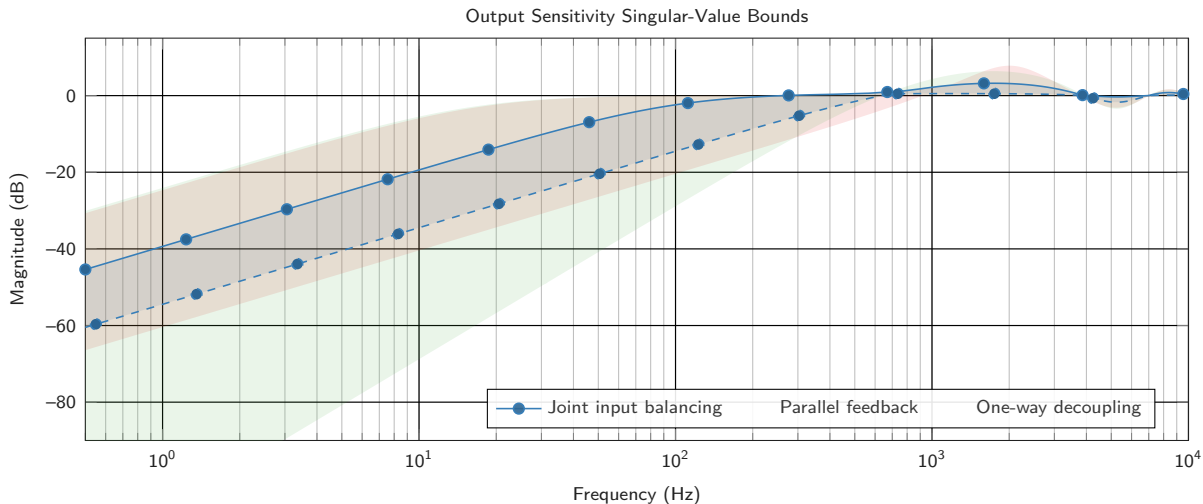
$$\left\| \left(I - \frac{\hat{\eta} \hat{\eta}^T}{\|\hat{\eta}\|_2^2} \right) \mathbf{S}_{(j\omega)} \frac{\hat{\eta}}{\|\hat{\eta}\|_2} \right\|_2$$

Complementary directions → dispersion direction:

$$\left\| \frac{\hat{\eta}^T}{\|\hat{\eta}\|_2} \mathbf{S}_{(j\omega)} \left(I - \frac{\hat{\eta} \hat{\eta}^T}{\|\hat{\eta}\|_2^2} \right) \right\|_2$$

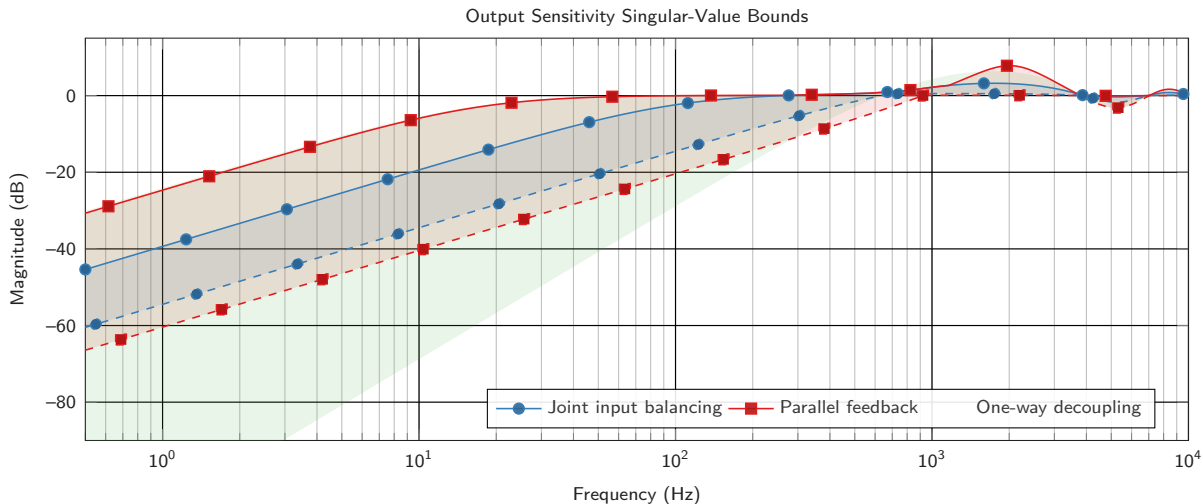


Sensitivity: Transfer Function Matrix from Disturbance $\mathbf{d}(j\omega)$ to Output $\mathbf{y}(j\omega)$

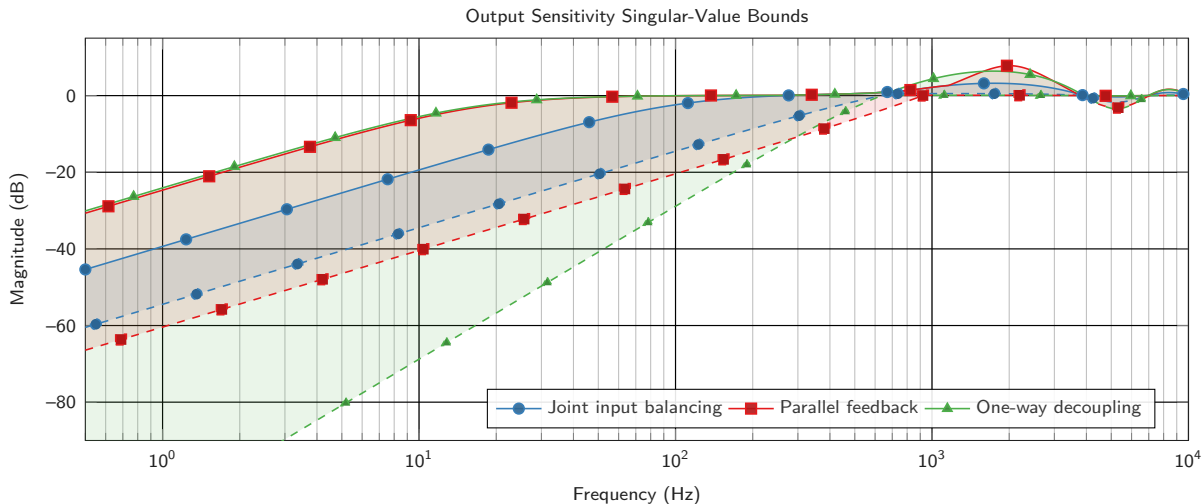


Solid: maximum singular value (least attenuation); Dashed: minimum singular value (strongest attenuation).

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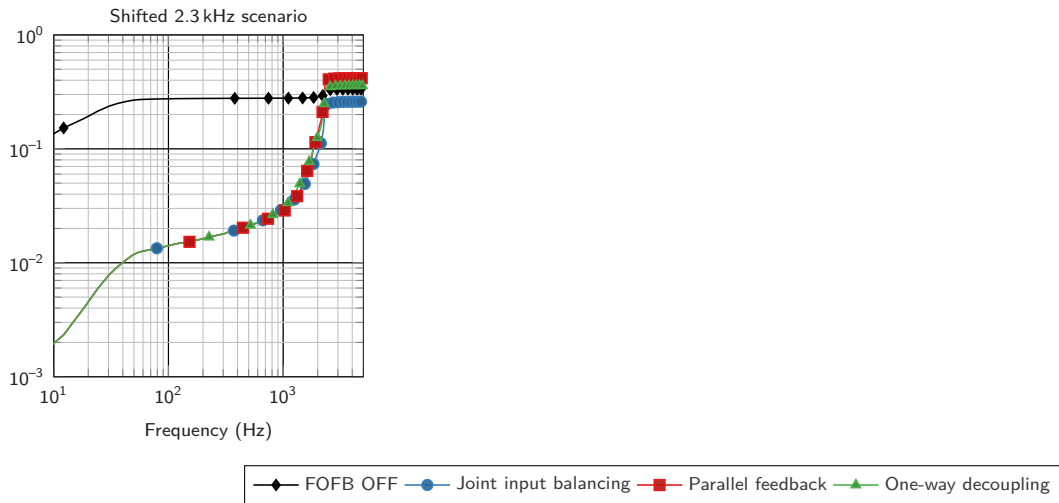


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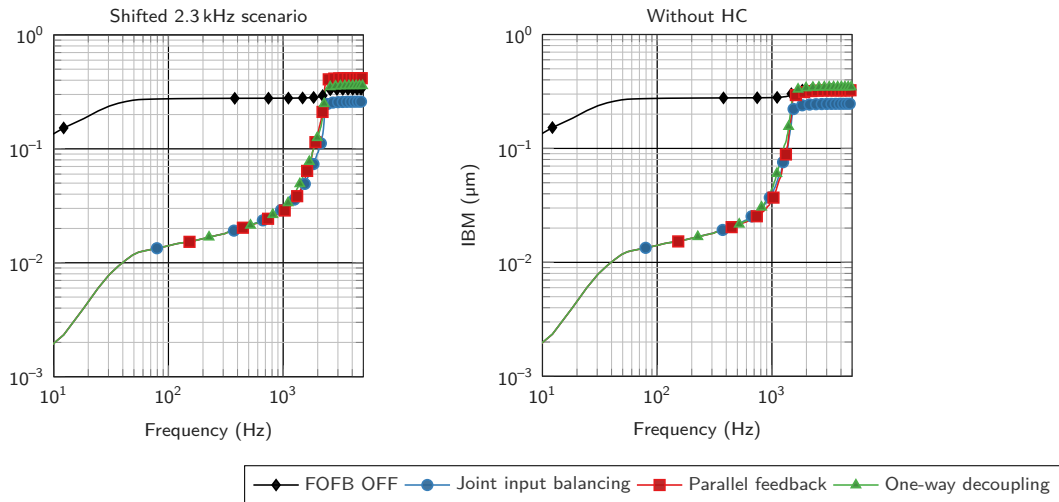
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Spatial RMS Integrated Beam Motion: Diamond-II Synthetic Disturbance



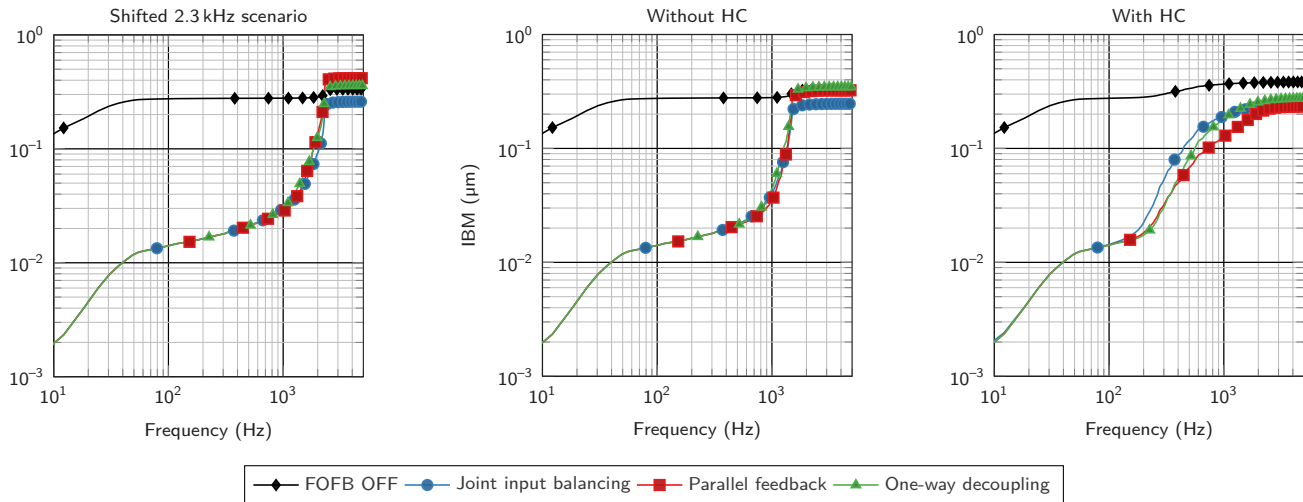
Detrended Welch IBM; dotted lines mark saved spectral peaks. Shifted case uses Diamond-II geometry, not a calibrated current-Diamond model.

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Discussion and Outlook

- ▶ Near-orthogonality of FOFB and RF actuator directions does not imply decoupled closed-loop behaviour

Future work:

Discussion and Outlook

- ▶ Near-orthogonality of FOFB and RF actuator directions does not imply decoupled closed-loop behaviour
- ▶ Control architectures considered:
 - ▶ **Parallel feedback**: doubles loop gain in $\hat{\eta}$ direction
 - ▶ **One-way decoupling**: doubles sensitivity slope along $\hat{\eta}$ direction
 - ▶ **SVD/GSVD-based control**: uniform attenuation

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 - ▶ **SVD/GSVD-based control**: uniform attenuation
- ▶ For equivalent control schemes: higher bandwidths result in reduced robustness

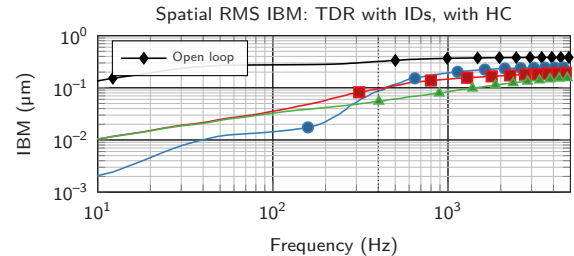
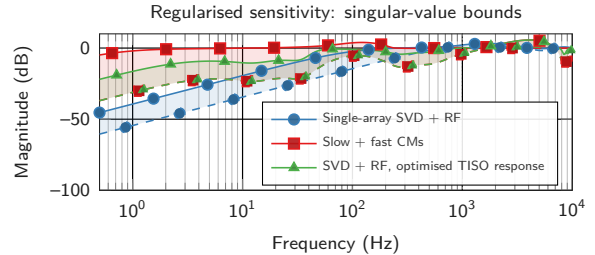
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Future work:

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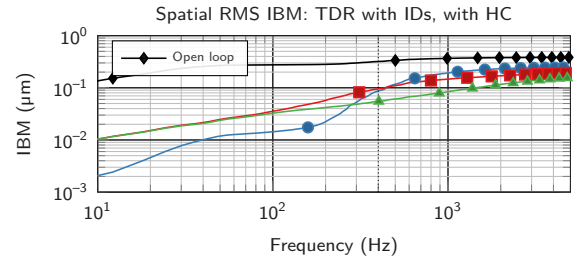
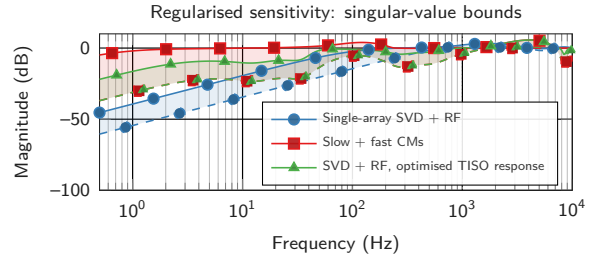


Discussion and Outlook

- ▶ Near-orthogonality of FOFB and RF actuator directions does not imply decoupled closed-loop behaviour
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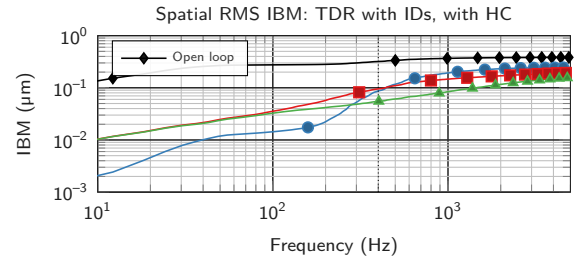
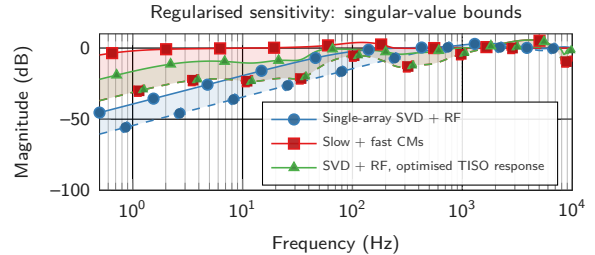


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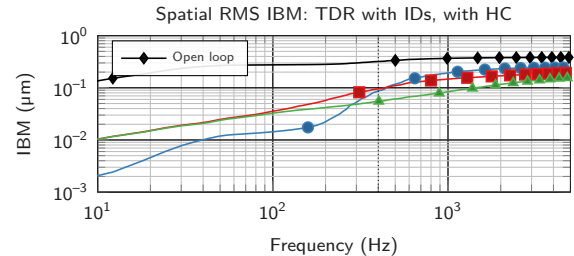
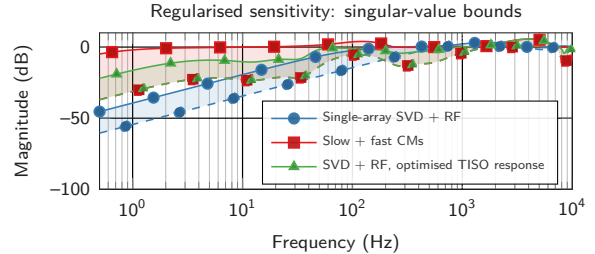


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- ▶ Experimental validation and eventual deployment on Diamond-II



Thank you!

With kind support of:



Dr. Lorraine Bobb
Diamond Light Source



Dr. Shohan Banerjee
Diamond Light Source



and



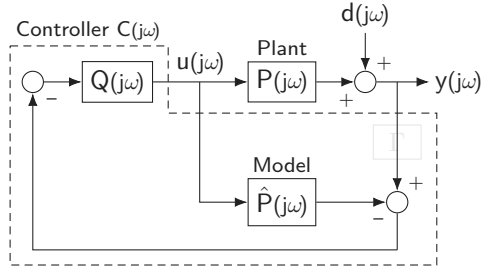
Grant number EP/X525777/1

idris.kempf@eng.ox.ac.uk

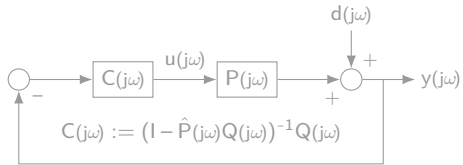
Applied Control Laboratory: <https://users.ox.ac.uk/~lady5906/>

Appendix

Internal Model Control for Systems with One Corrector Type (Diamond)



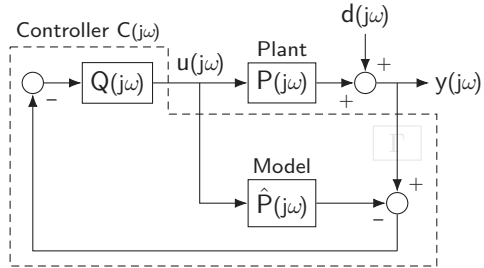
$$P(j\omega) := Rg(j\omega), \quad \hat{P}(j\omega) := \bar{R}\bar{g}(j\omega)$$



$$C(j\omega) := (I - \hat{P}(j\omega)Q(j\omega))^{-1}Q(j\omega)$$

⇒ M. Morari and E. Zafiriou, *Robust Process Control*, 1989.

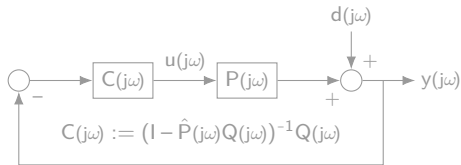
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1. Choose $T(j\omega)$ and invert plant:

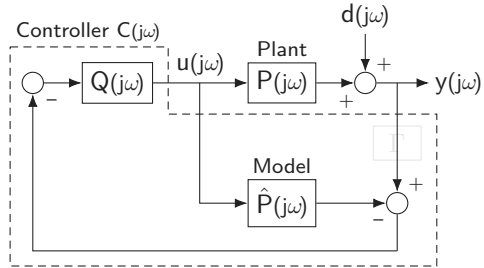
$$Q(j\omega) = \hat{P}^\dagger(j\omega) T(j\omega)$$



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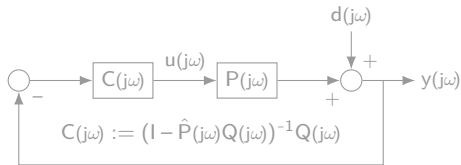
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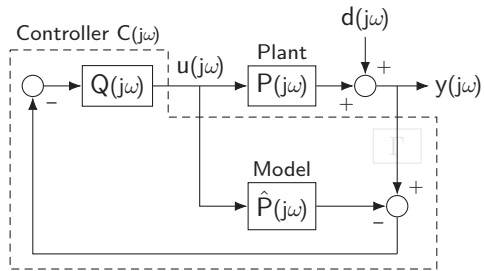
$$y(j\omega) = (1 - T(j\omega)) d(j\omega)$$



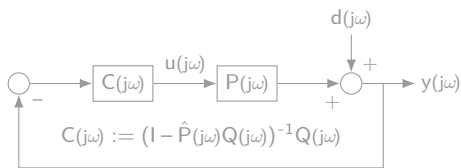
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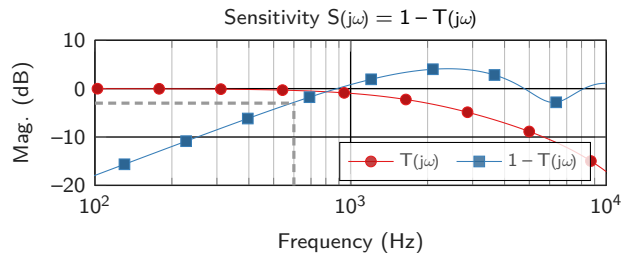


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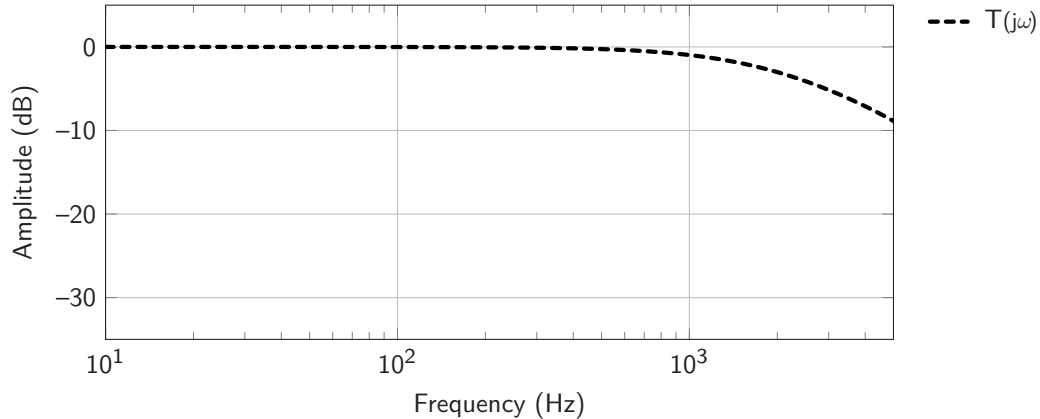
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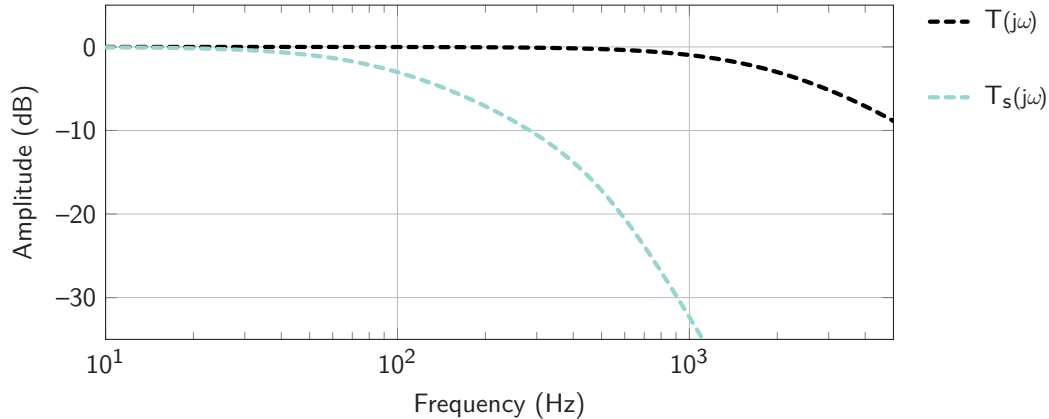


Mid-Ranging Control: Split Control Effort $T(j\omega)$ between Slow and Fast Correctors



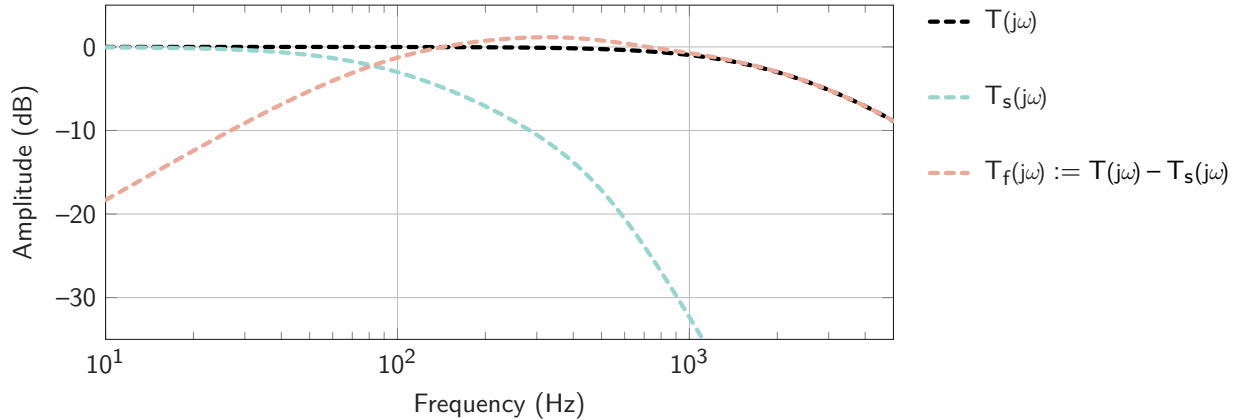
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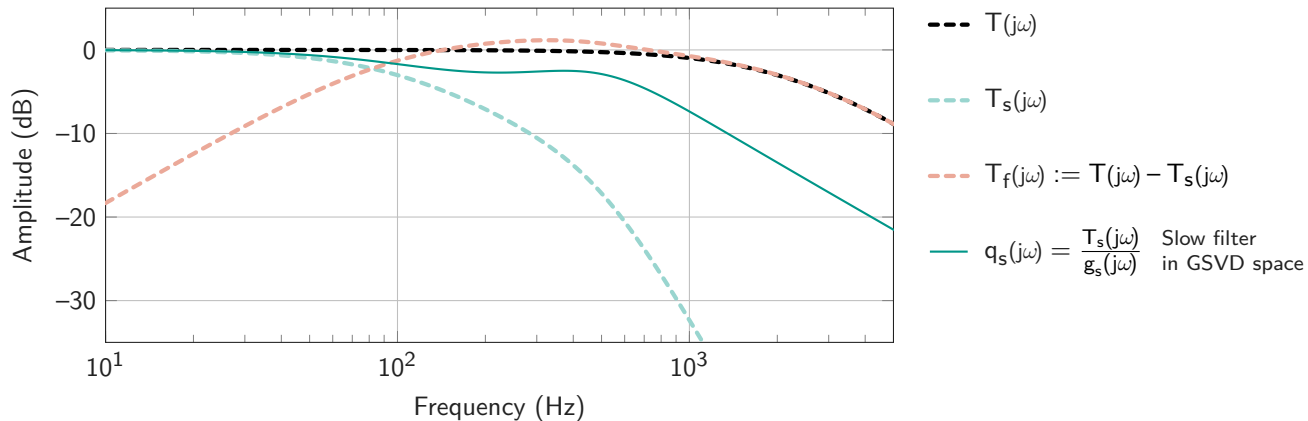
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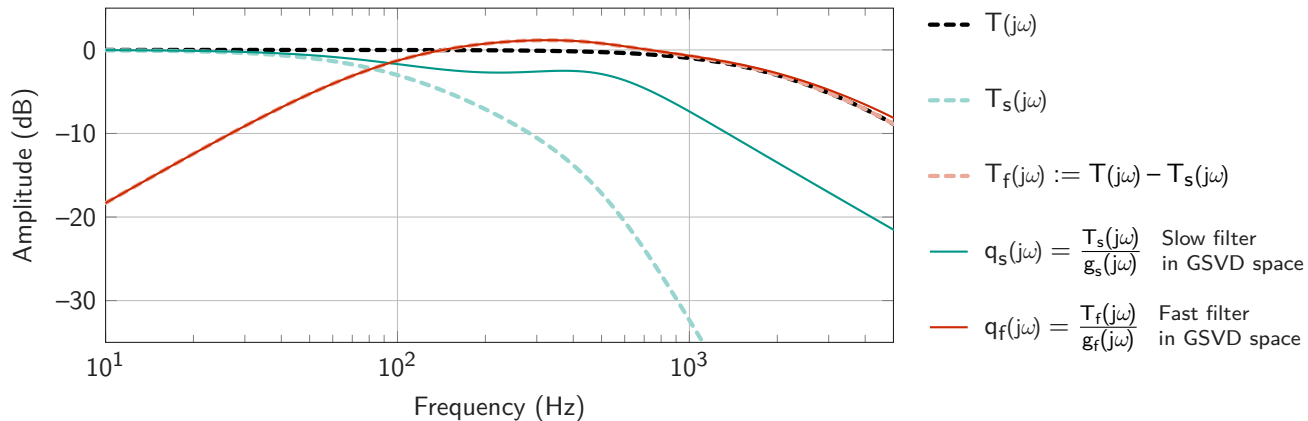
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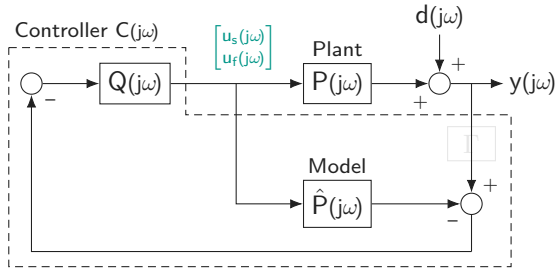
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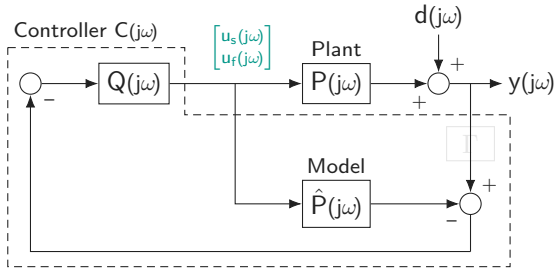
Internal Model Control for Systems with Slow and Fast Correctors



$$P(j\omega) := [R_s g_s(j\omega) \quad R_f g_f(j\omega)] := [P_s(j\omega) \quad P_f(j\omega)], \quad \hat{P}(j\omega) := \dots$$

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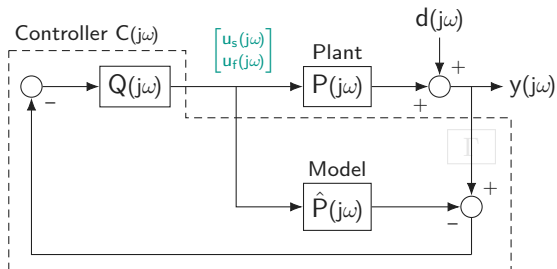
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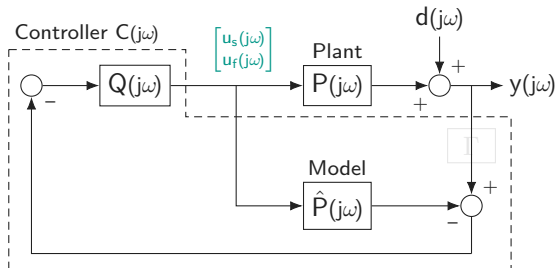
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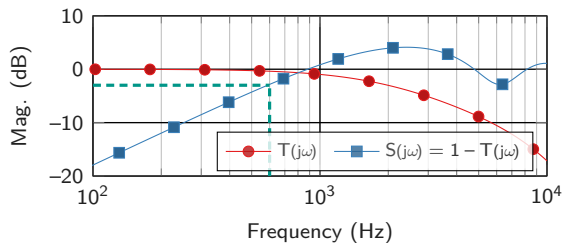
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3. Add Υ_f if $n_f < n_s$

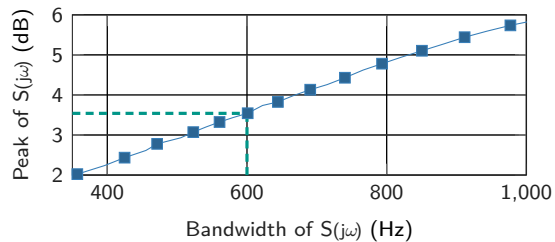
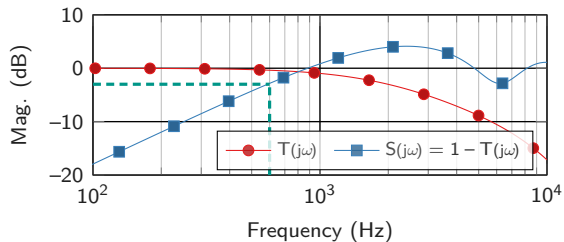
Choosing the Closed-Loop Bandwidth for Diamond-II

Bandwidth $\uparrow \Rightarrow$ Peak of $S(j\omega) \uparrow \Rightarrow$ Instability, disturbance amplification



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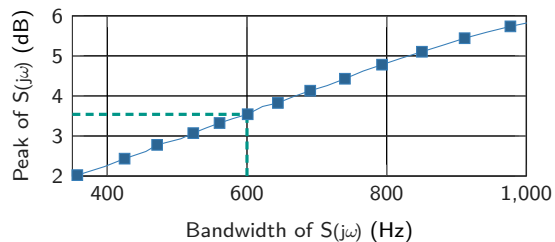
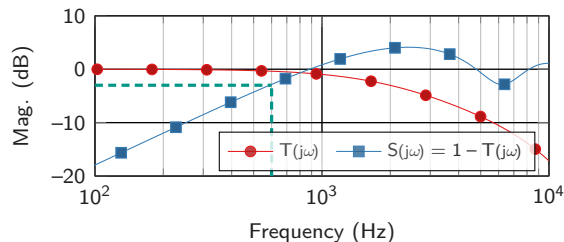


Limiting factors for Diamond-II:

- Time delay
- Complexity of $g_{(\cdot)}(j\omega)$
- Slow corrector bandwidth

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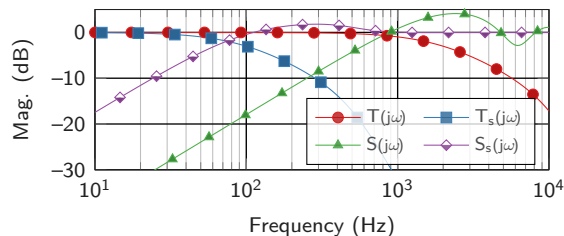
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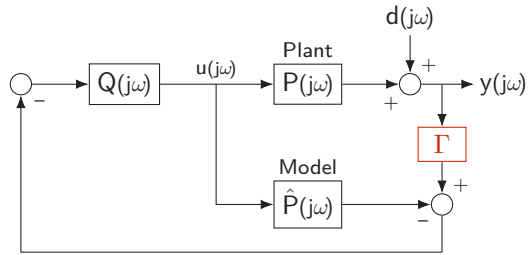
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Preliminary Diamond-II Design



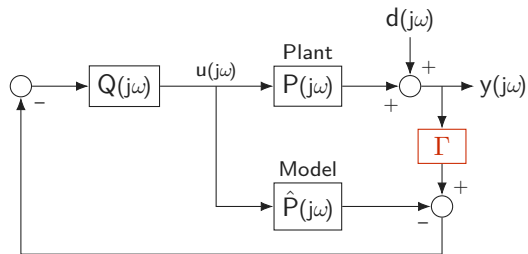
Tikhonov Regularisation: Avoid Large Control Inputs for Slow and Fast Correctors



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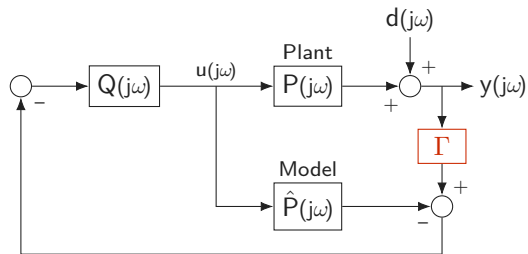
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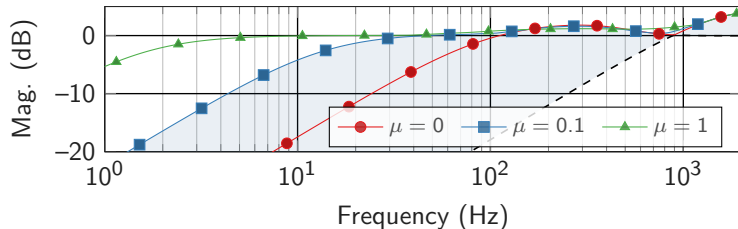
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Overall Attenuation of $d(j\omega)$ – Vertical



✓ Mainly affects higher-order modes

✓ Disturbance concentrated in low-order modes